Variational solution to hemodynamic and perfusion response estimation from ASL fMRI data

Aina Frau-Pascual, Florence Forbes, Philippe Ciuciu

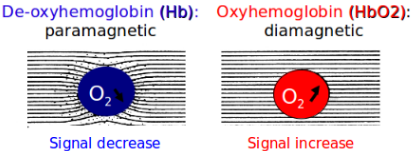
June, 2015

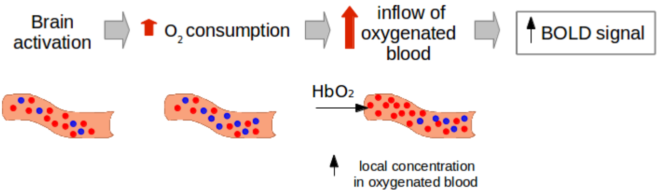
BOLD: Qualitative functional MRI

- Blood Oxygen Level Dependent [Ogawa et al, PNAS 1990]

What does BOLD contrast really measure?

BOLD measures the ratio of oxy- to deoxy-hemoglobin in the blood



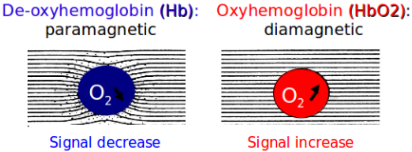


BOLD: Qualitative functional MRI

- Blood Oxygen Level Dependent [Ogawa et al, PNAS 1990]

What does BOLD contrast really measure?

BOLD measures the ratio of oxy- to deoxy-hemoglobin in the blood



BOLD signal → % signal change → CBF cerebral blood flow + CBV cerebral blood volume + oxygen consumption

ASL: Quantitatively imaging cerebral perfusion

- Arterial Spin Labeling [Williams et al, PNAS 1992]

WHAT?

- Cerebral perfusion: Delivery of nutritive blood to the brain tissue capillary bed

WHY?

- Quantification is important: eg. perfusion altered in various diseases (stroke, tumors)

ASL	BOLD
✓ direct quantitative measure	✗ indirect measure
✓ cerebral blood flow	✗ mix of parameters
✗ low SNR	✓ higher SNR (» ASL)

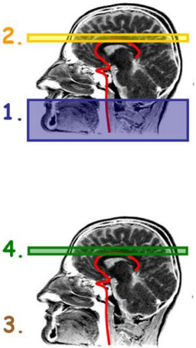
Arterial Spin Labeling data acquisition

Tag image

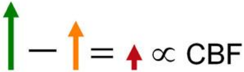
Tag inflowing arterial blood
by magnetic inversion

Control image

Repeat experiment without
labeling inflowing blood



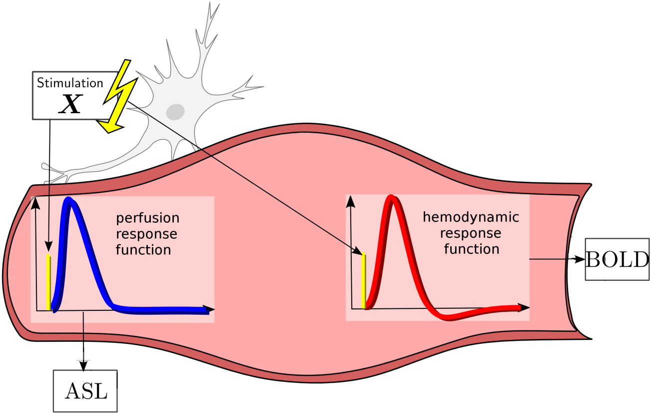
Control Image (4) - Tag Image (2)



Ref: http://fmri.research.umich.edu/research/main_topics/asl.php

Statistical analysis of ASL fMRI data

ASL data contain both hemodynamic & perfusion components



CBF

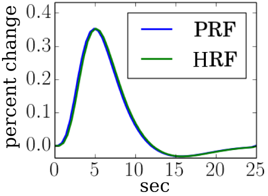
CBF + blood volume
+ oxygen consumption

Statistical analysis of ASL fMRI data

- GLM

  Unique fixed canonical hemodynamic response function (HRF)
  [Hernandez-Garcia et al, 10,
  Mumford et al, 06]

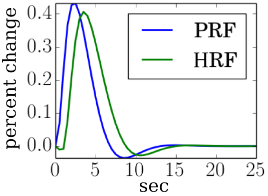
  Inaccurate PRF shapes



- Joint Detection-Estimation (JDE)

  Separate estimation of 2 response functions (HRF & PRF)
  Use of MCMC methods
  [Vincent et al, 13,
  Frau-Pascual et al, 14]

  Computationally very expensive



GOAL

Providing an efficient solution to hemodynamic and perfusion response estimation from ASL fMRI data

Based on:

- Variational Expectation-Maximization [Chaari et al, 12]
  - Acceptable computational times
- Physiological prior information

ASL signal model

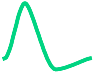
ASL signal = perfusion baseline + task-related perfusion + task-related BOLD + drift term + noise term

$$\boldsymbol{y}_j = \alpha_j \boldsymbol{w} + \sum_{m=1}^{M} \left(c_j^m \boldsymbol{W} \boldsymbol{X}^m \boldsymbol{g} + a_j^m \boldsymbol{X}^m \boldsymbol{h} \right) + \boldsymbol{P} \boldsymbol{\ell}_j + \boldsymbol{b}_j$$

$\boldsymbol{g}$

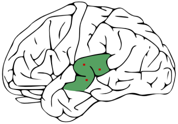
perfusion response function (PRF)

$\boldsymbol{h}$

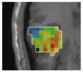


hemodynamic response function (HRF)

parcel-wise

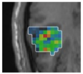


$\boldsymbol{c}$



perfusion response levels (PRLs)

$\boldsymbol{a}$

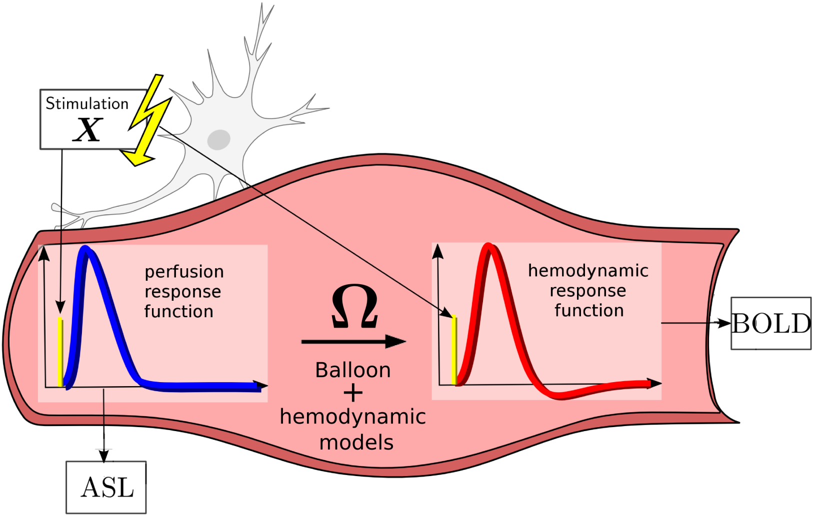


hemodynamic response levels (HRLs)

voxel-wise

$\boldsymbol{q}$ labels (active/non-active)

Physiological prior



Variational Expectation Maximization

- Expectation Maximization.

$$\textbf{E-step: } \tilde{p}^{(r)} = \arg\max_{\tilde{p}} \; F(\tilde{p}, \boldsymbol{\theta}^{(r)})$$

$$\textbf{M-step: } \boldsymbol{\theta}^{(r+1)} = \arg\max_{\boldsymbol{\theta}} \; F(\tilde{p}^{(r)}, \boldsymbol{\theta})$$

  being

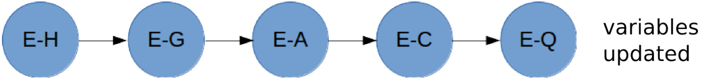
$$F(\tilde{p}, \boldsymbol{\theta}) = \mathrm{E}_{\tilde{p}}\left[\log p(\boldsymbol{y}, \boldsymbol{a}, \boldsymbol{h}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{q} \; ; \; \boldsymbol{\theta})\right] \underbrace{- \mathrm{E}_{\tilde{p}}\left[\log \tilde{p}(\boldsymbol{a}, \boldsymbol{h}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{q})\right]}_{\text{entropy of } \tilde{p}}$$

- Variational EM: class of probability distributions restricted to the set of distributions that satisfy

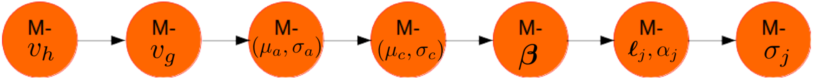
$$\tilde{p}(\boldsymbol{a}, \boldsymbol{h}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{q}) = \tilde{p}_a(\boldsymbol{a}) \; \tilde{p}_h(\boldsymbol{h}) \; \tilde{p}_c(\boldsymbol{c}) \; \tilde{p}_g(\boldsymbol{g}) \; \tilde{p}_q(\boldsymbol{q})$$

VEM steps (1)

The E-step becomes an approximate E-step that can be further decomposed into five stages updating the different variables:



The M-step can also be divided into separate steps:



VEM steps (2)

The E-step become:

$$\textbf{E-H-step:} \quad \tilde{p}_h = \operatorname*{arg\,max}_{\tilde{p}_h \in \mathcal{D}_H} F(\tilde{p}_a\, \tilde{p}_h\, \tilde{p}_c\, \tilde{p}_g\, \tilde{p}_q; \theta)$$

$$\textbf{E-G-step:} \quad \tilde{p}_g = \operatorname*{arg\,max}_{\tilde{p}_g \in \mathcal{D}_G} F(\tilde{p}_a\, \tilde{p}_h\, \tilde{p}_c\, \tilde{p}_g\, \tilde{p}_q; \theta)$$

and similar for the rest of the variables.
The M-step can also be divided into separate steps:

$$\begin{aligned}
\boldsymbol{\theta} = \operatorname*{arg\,max}_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} \Big[& \mathrm{E}_{\tilde{p}_a \tilde{p}_c}\big[\log p(\boldsymbol{y} \,|\, \boldsymbol{a}, \tilde{\boldsymbol{h}}, \boldsymbol{c}, \tilde{\boldsymbol{g}}; \boldsymbol{\alpha}, \boldsymbol{\ell}, \boldsymbol{\sigma}^2)\big] \\
& + \mathrm{E}_{\tilde{p}_a \tilde{p}_q}\big[\log p(\boldsymbol{a} \,|\, \boldsymbol{q}; \boldsymbol{\mu}_a, \boldsymbol{\sigma}_a)\big] + \log p(\tilde{\boldsymbol{h}}; v_h) \\
& + \mathrm{E}_{\tilde{p}_c \tilde{p}_q}\big[\log p(\boldsymbol{c} \,|\, \boldsymbol{q}; \boldsymbol{\mu}_c, \boldsymbol{\sigma}_c)\big] + \log p(\tilde{\boldsymbol{g}}; v_g) \\
& + \mathrm{E}_{\tilde{p}_q}\big[\log p(\boldsymbol{q}; \boldsymbol{\beta})\big] \Big]
\end{aligned}$$

Constraints on $\boldsymbol{h}$ and $\boldsymbol{g}$

We can constraint the search to pointwise estimates $\tilde{\boldsymbol{h}}$ and $\tilde{\boldsymbol{g}}$ by replacing the probabilities on $\boldsymbol{h}$ and $\boldsymbol{g}$ by Dirac functions:

$$\tilde{p} = \tilde{p}_a \, \delta_{\tilde{\boldsymbol{h}}} \, \tilde{p}_c \, \delta_{\tilde{\boldsymbol{g}}} \, \tilde{p}_q$$

so that, for example for H, the E-H step

$$\tilde{p}_h = \arg\max_{\tilde{p}_h \in \mathcal{D}_H} F(\tilde{p}_a \, \tilde{p}_h \, \tilde{p}_c \, \tilde{p}_g \, \tilde{p}_q; \theta)$$

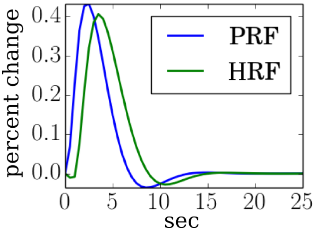
$$\text{becomes} \qquad \tilde{h} = \arg\max_{\tilde{h}} F(\tilde{p}_a \delta_{\tilde{h}} \tilde{p}_c \delta_{\tilde{g}} \tilde{p}_q; \theta)$$

This facilitates the inclusion of constraints on $\boldsymbol{h}$ and $\boldsymbol{g}$ like $\|\boldsymbol{h}\|_2^2 = 1$ and $\|\boldsymbol{g}\|_2^2 = 1$.

Simulation results

Artificial data generation

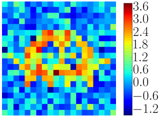
- Repetition time: TR $= 3$ s
- Number of scans: $N = 288$
- White noise $\boldsymbol{b}_j \sim \mathcal{N}(0, 2)$
- Response functions simulated with physiological model [Friston et al, 00]



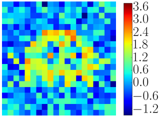
- Fast event-related paradigm:

  mean ISI$= 5$ s.
- 1 experimental condition 20×20 binary activation label maps:

  hemodyn. maps $\sim \mathcal{N}(2.2, 0.3)$



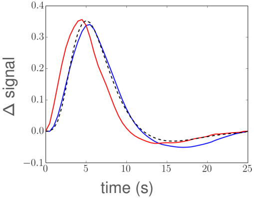
perfusion maps $\sim \mathcal{N}(1.6, 0.3)$



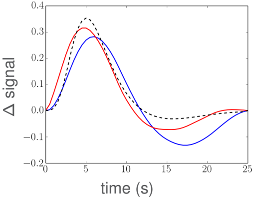
Simulation results: Low SNR scenario, TR $= 3$s

- Response function

SNR
2.4 dB



SNR
0.5 dB



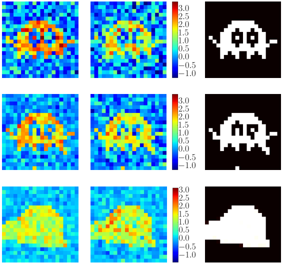
- Response levels

hemodyn. maps | perfusion maps | activation states

ground truth

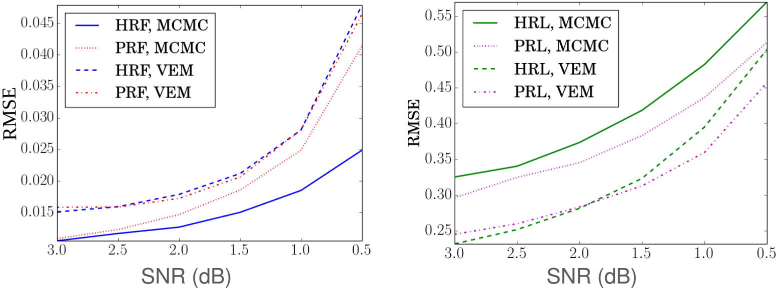
SNR 2.4 dB

SNR 0.5 dB



Simulation results: Low SNR scenario, TR $= 3$s

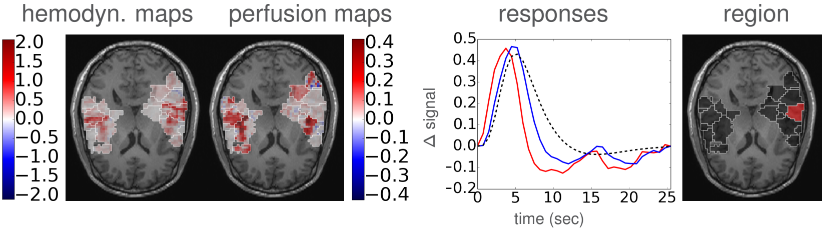
Comparison to MCMC solution of joint detection estimation (JDE):



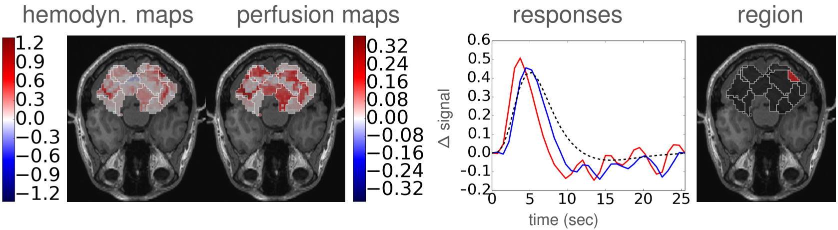
Real data

Paradigm: fast event-related design (mean ISI = 5.1s.), with 60 auditory and visual stimuli

Auditory cortex



Visual cortex



Conclusions

- Jointly detecting activity and estimating hemodynamic and perfusion responses from functional ASL data
- It facilitates the inclusion of additional information

Future directions

- Performance optimization
- Investigation of other constraints

Thanks for your attention