Estimation of Mars surface physical properties from hyperspectral images using the SIR method

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Source: ESA

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INRIA RHÔNE-ALPES

Outline

I. Context
- Hyperspectral image data
- Inverse problem

II. Dimension reduction
- PCA
- SIR

III. Regularization and estimation
- Zhong et al., 2005
- Tikhonov

IV. Validation on simulations

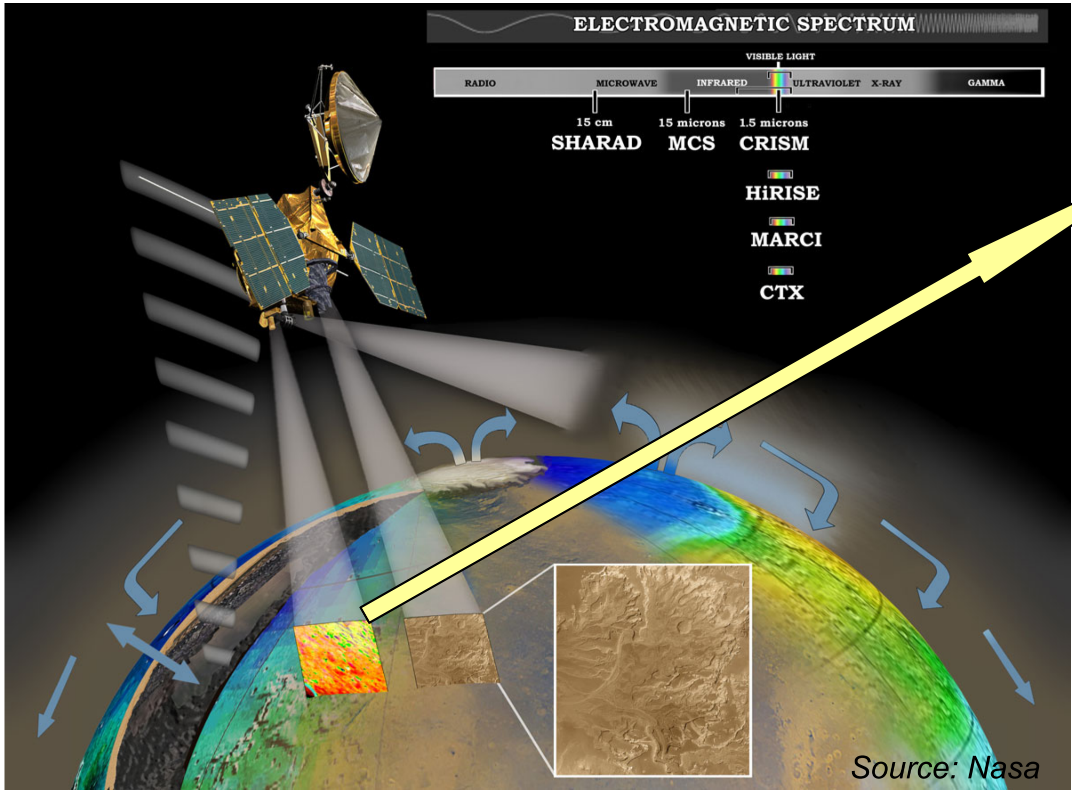
V. Application to the south polar cap of Mars

VI. Conclusion and future work

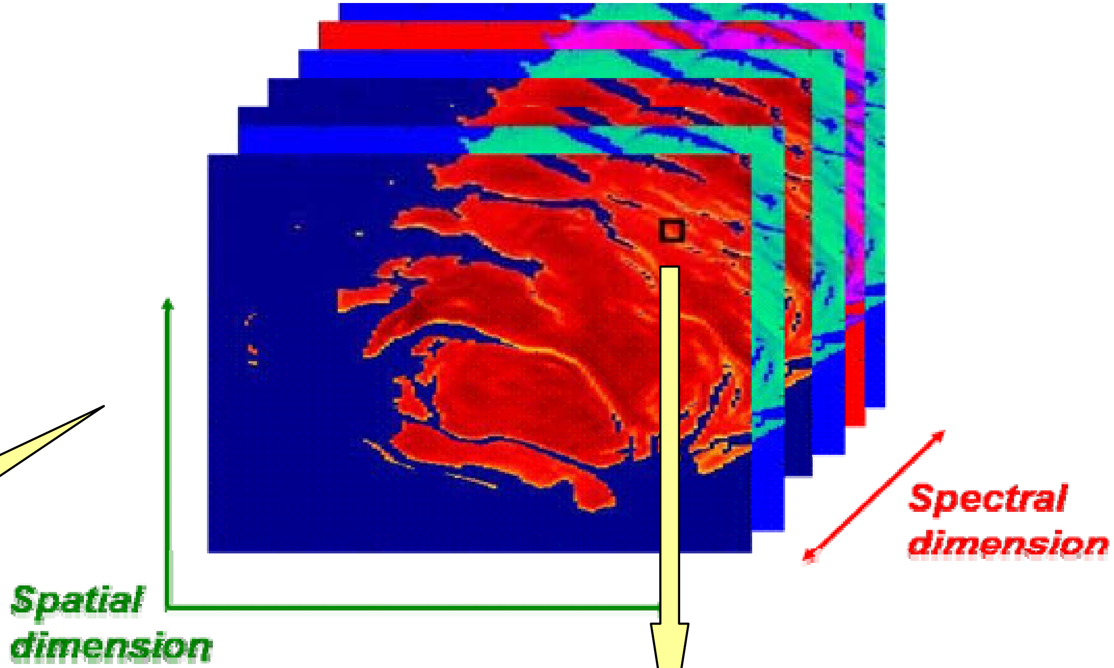
Outline

Introduction

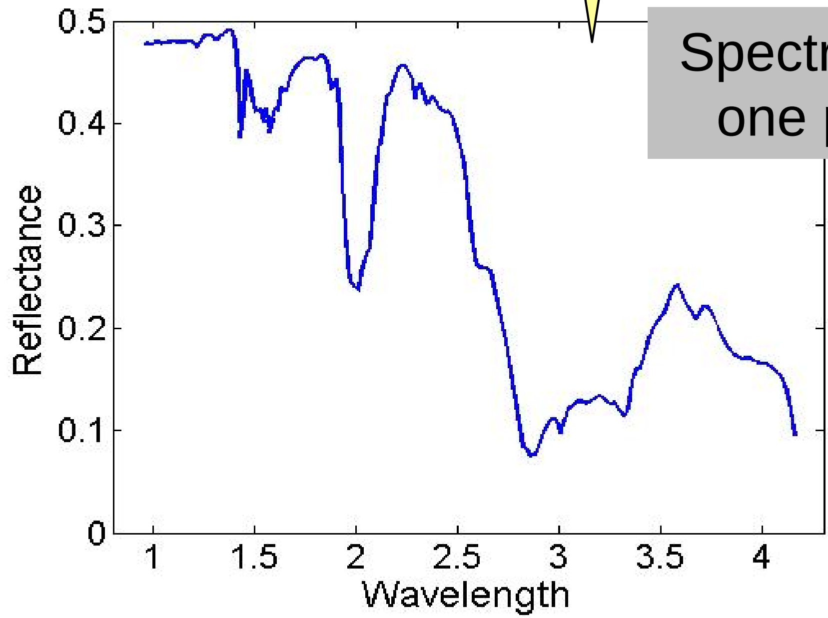
Spectrometer



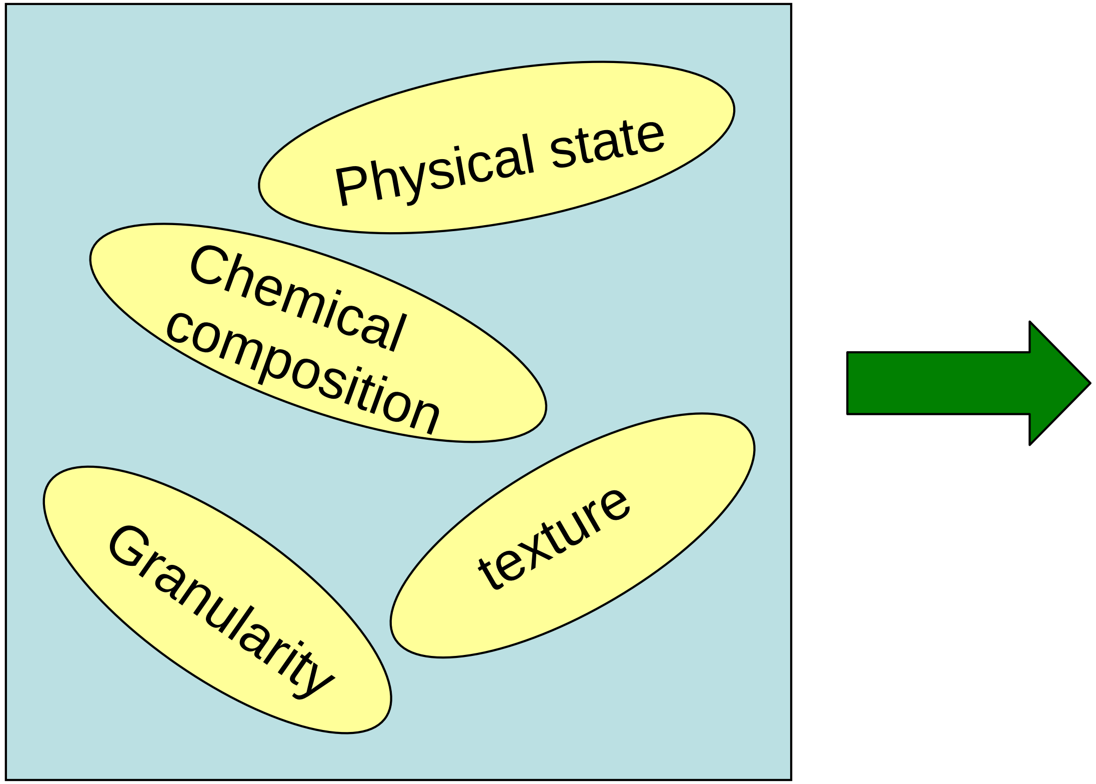
Hyperspectral cube

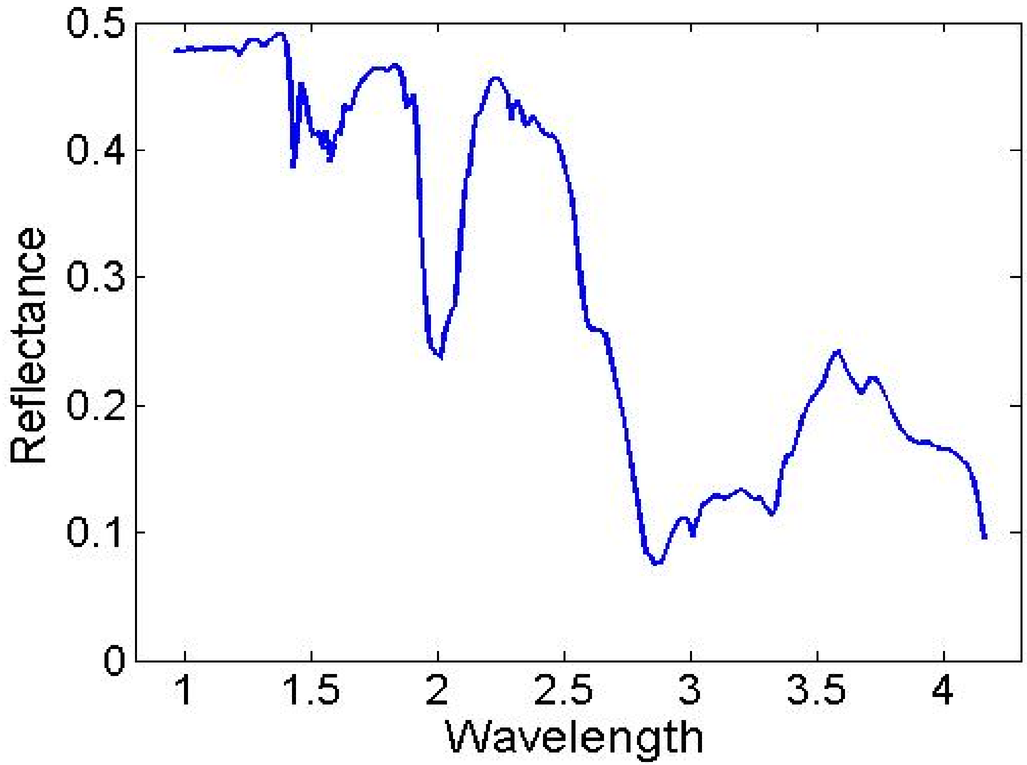


Spectrum at one pixel



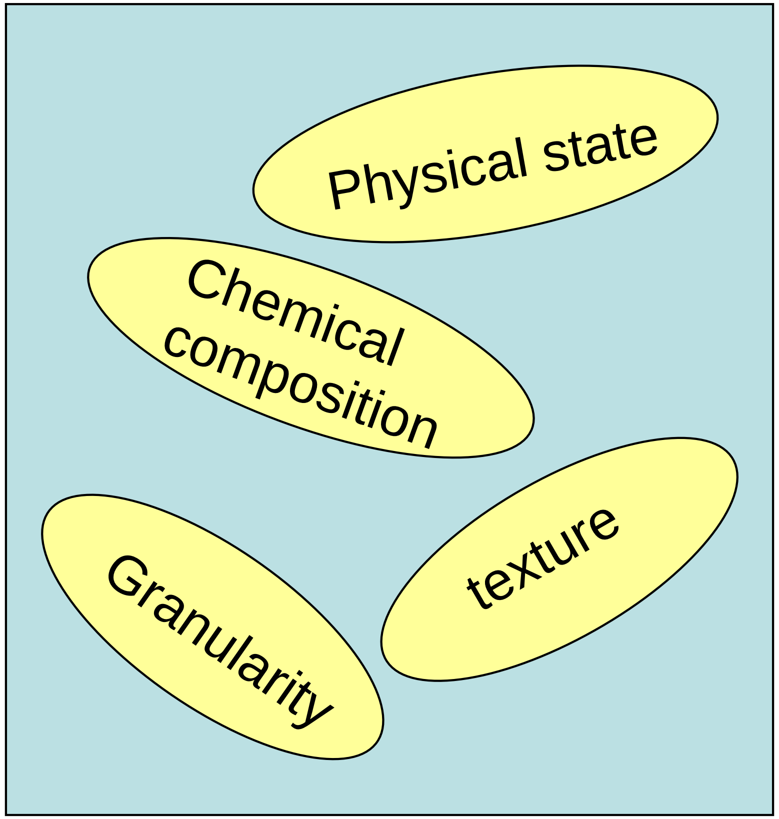
Radiative transfer model

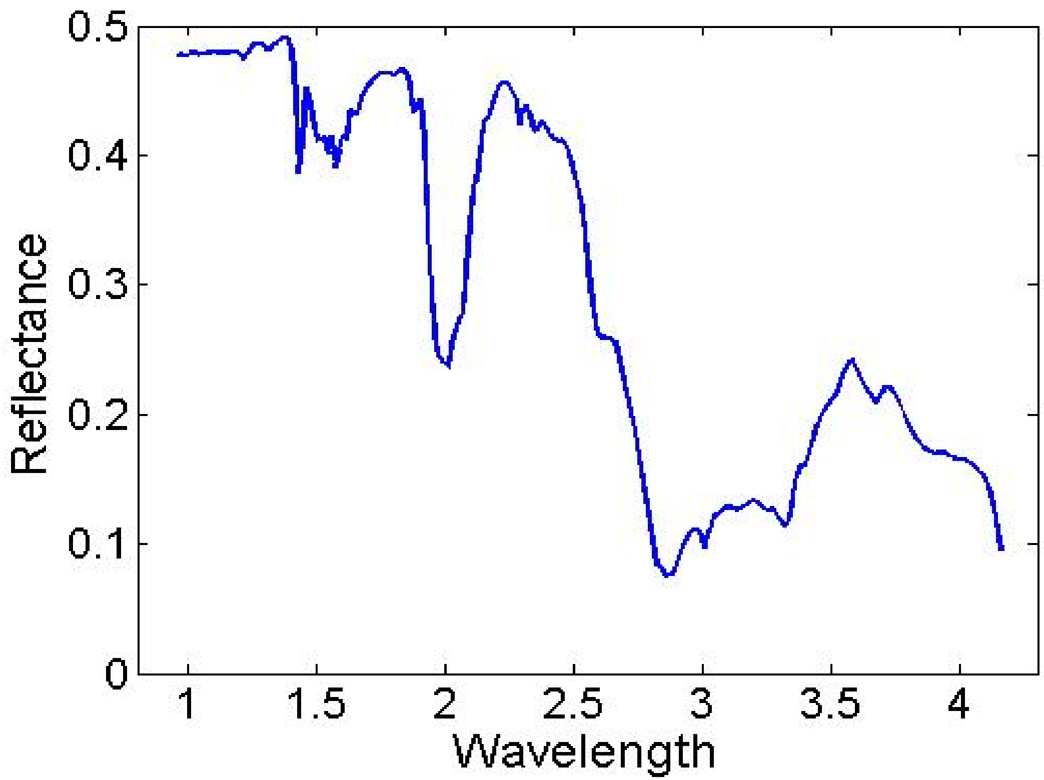




RADIATIVE TRANSFER MODEL: evaluates direct link between parameters and spectra. Allows the construction of a training data

Inverse problem

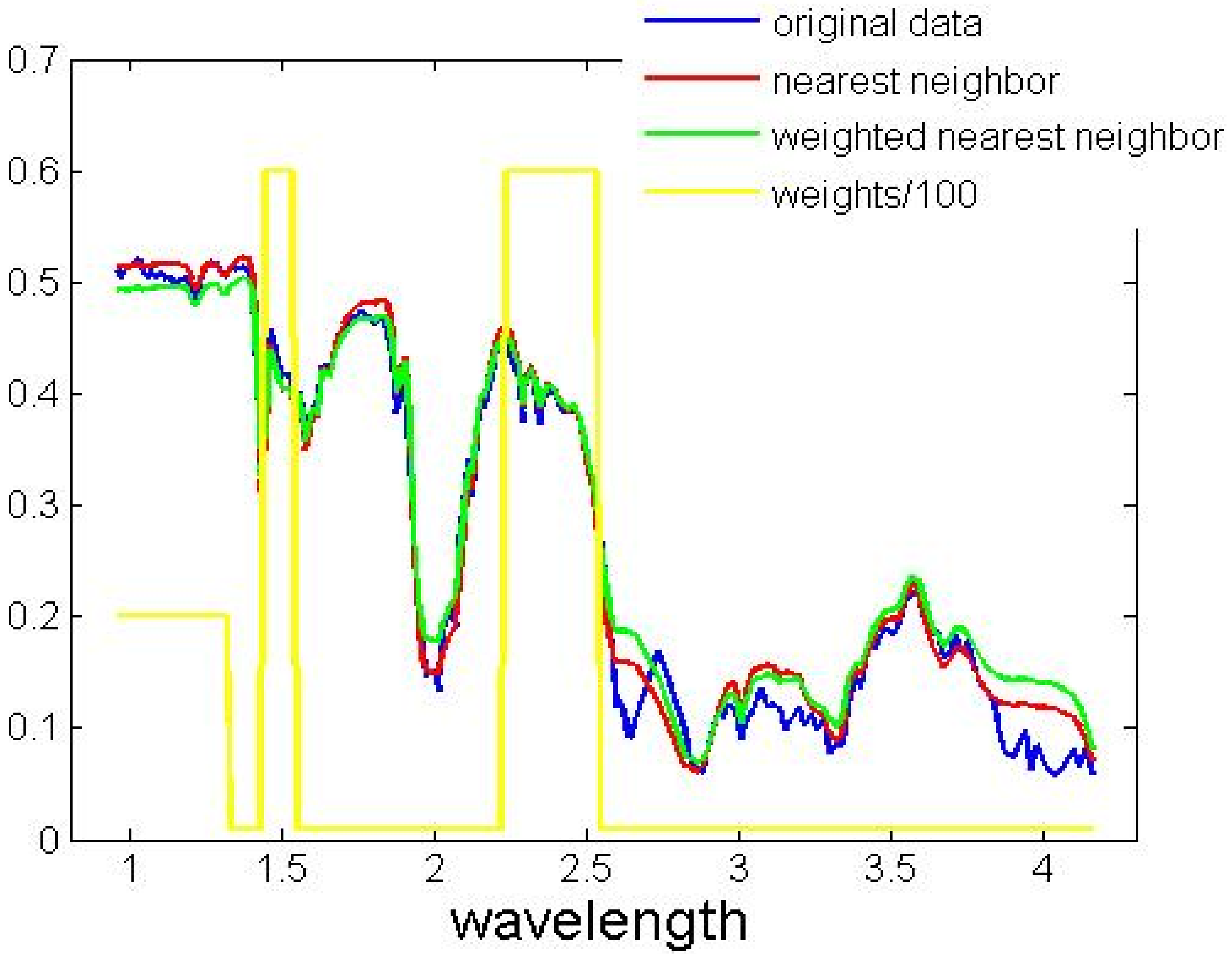




INVERSE PROBLEM: evaluates the properties of atmospheric and surface materials from the spectra

Usual methods

- Nearest neighbor
- Weighted Nearest neighbor



Aim

- To establish functional relationships between:
  - Spectra $x \in \mathbb{R}^p$ (p=184) from Mars Express mission
  - Physical parameter $y \in \mathbb{R}$: proportion of water, proportion of dust, grain size…
  - Construct *f* in order to estimate parameters:

$$f : \mathbb{R}^p \longrightarrow \mathbb{R}$$

$$x \longrightarrow y$$

Difficulties

- Curse of dimensionality (184 wavelengths): dimension of *x* has to be reduced
- Find projection axis $a \in \mathbb{R}^p$ (here, only the first axis will be retained)
- Instead of estimating f such as $y = f(x)$, we will suppose there exists $g : \mathbb{R} \to \mathbb{R}$ exists such that:

$$y = g(< a, x >, \varepsilon)$$

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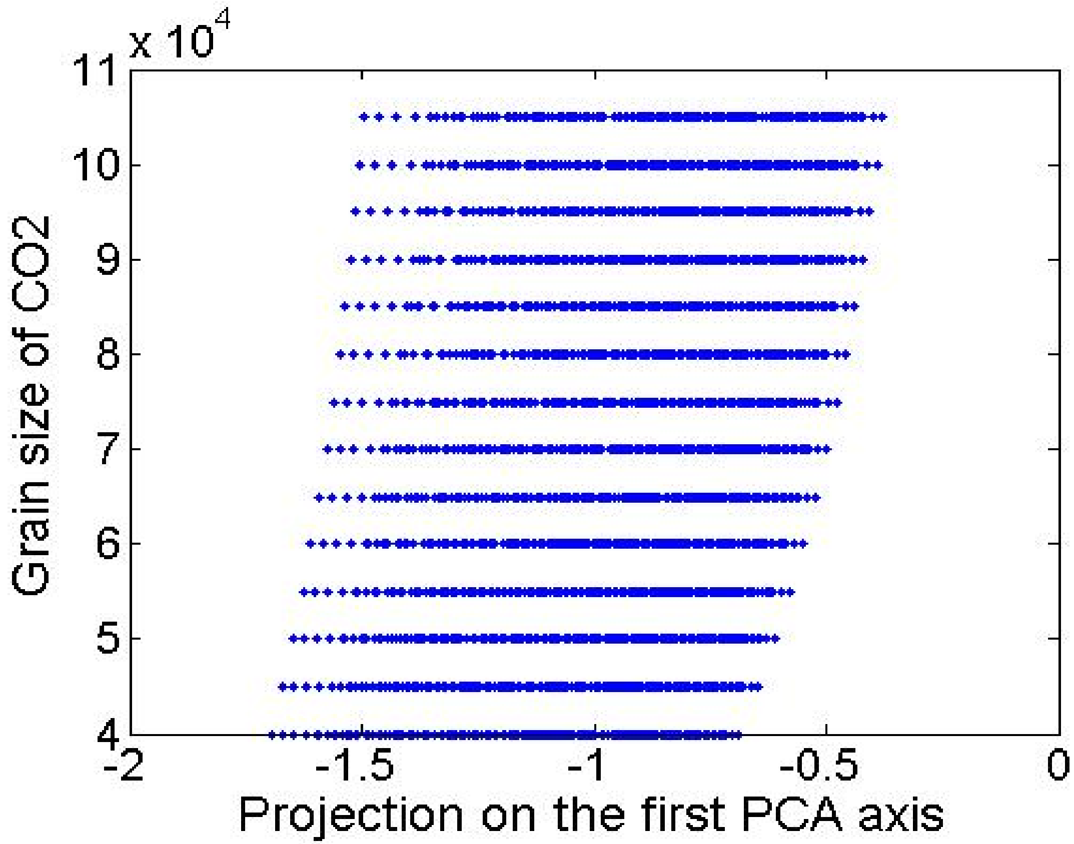
Principal component analysis

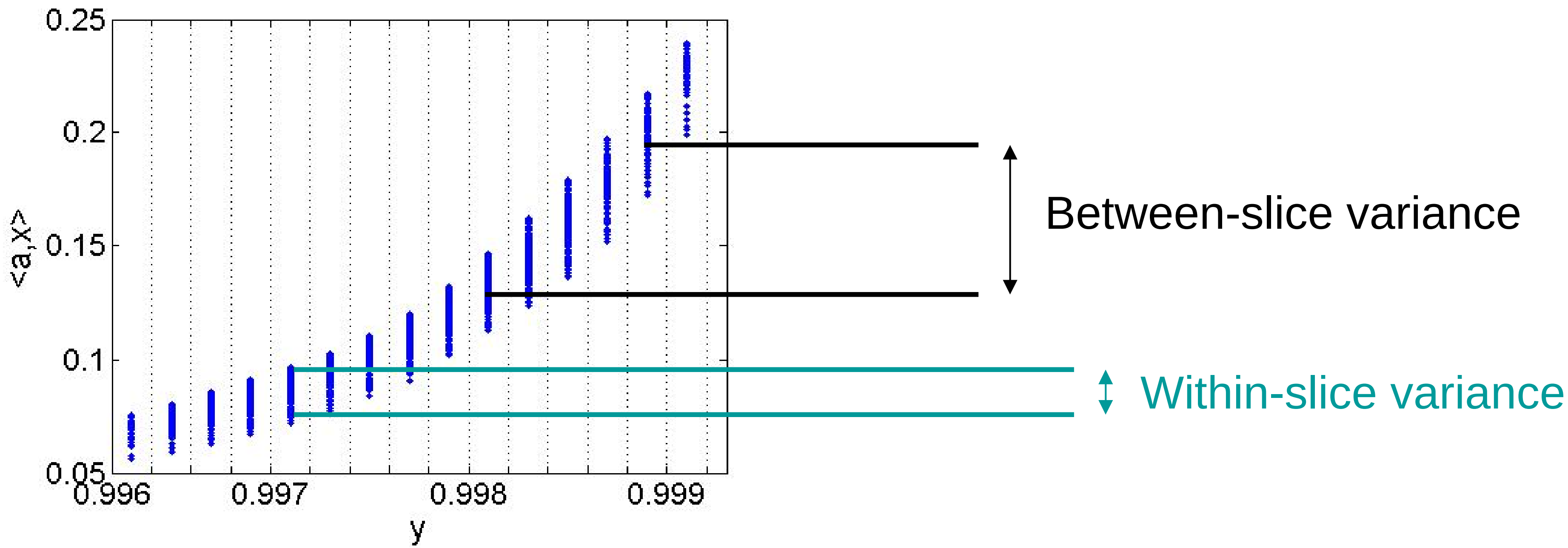
- **Maximizes the variance** of the projections of the observations *x*
- Does not take into account *y*



Sliced inverse regression

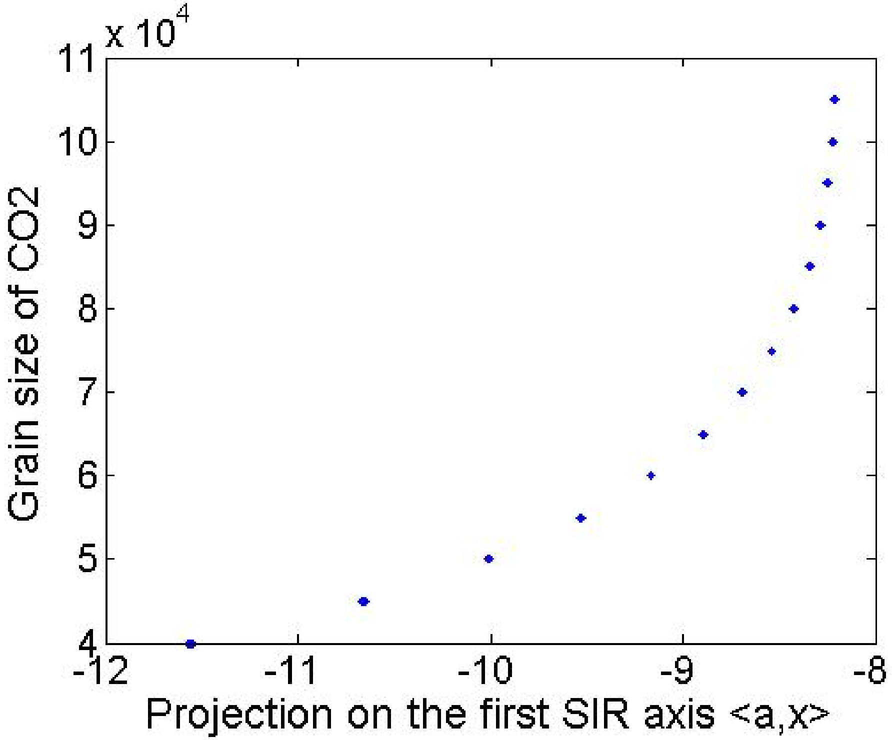
- Proposed by Li (1991)
- Maximizes the between-slice variance of projections
- PCA of E(*Z*/*Y*) with $Z = \Sigma^{-1/2} X$
- Eigenvectors of $\Sigma^{-1}\Gamma$ with $\Gamma = \mathrm{var}(E(X/Y))$

$$\Sigma = \mathrm{var}(X)$$

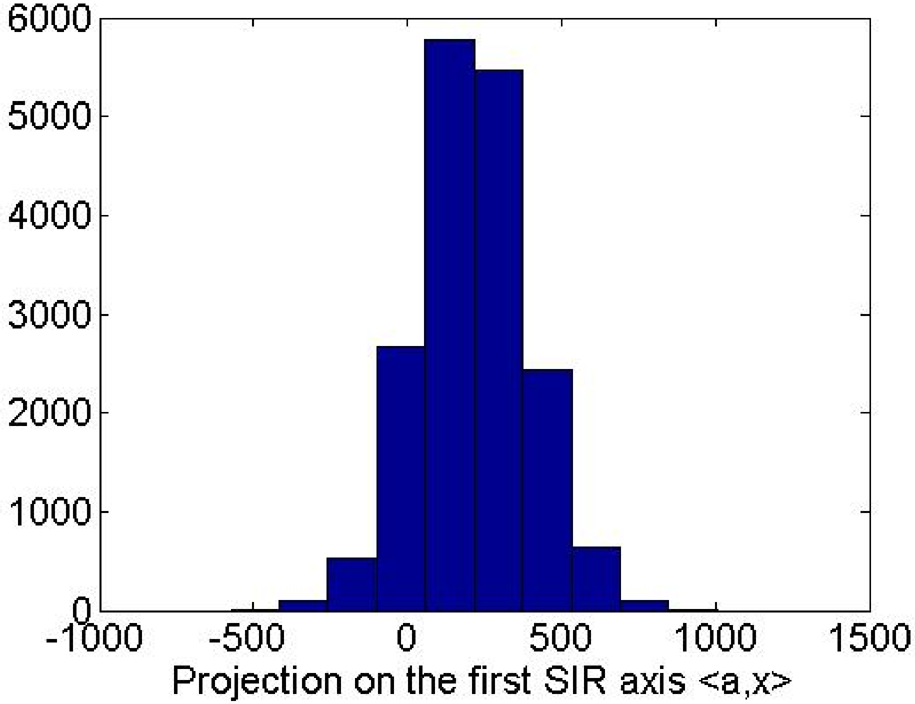


Application of SIR

TRAINING DATA



OBSERVED DATA



Problem

- Covariance matrix is ill-conditioned
  - Bad estimations of the directions
  - Sensitivity to noise
- Can be solved using regularization

DATA (x , y)

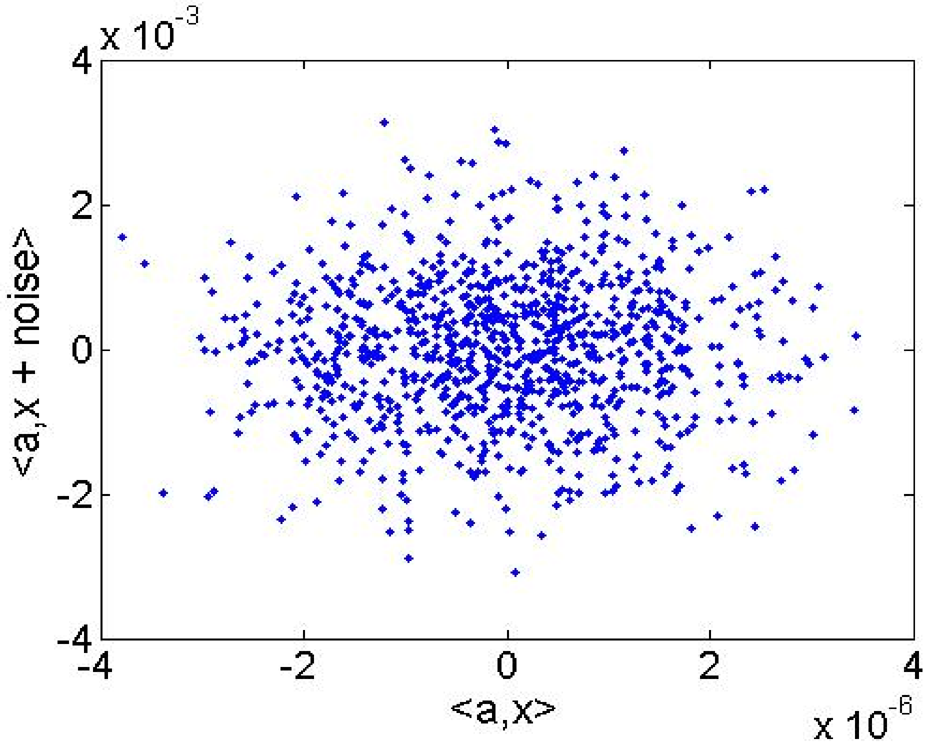
⇩

SLICED INVERSE REGRESSION

⇩

AXIS a

< a, x > < a , x + noise >



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Regularization (1)

- Usual SIR:
  - eigenvectors of $\Sigma^{-1}\Gamma$
- Regularized SIR:
  - Zhong et al., 2005: eigenvectors of

$$\left(\Sigma+\lambda Id\right)^{-1}\Gamma$$

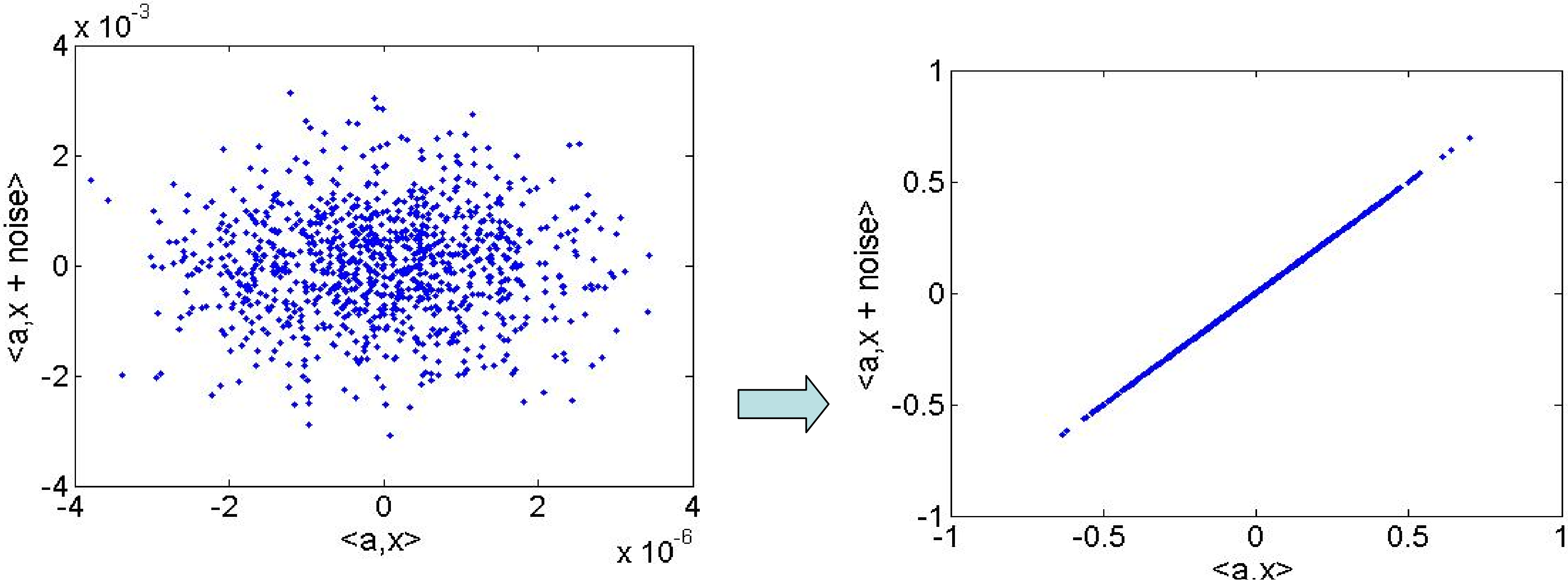
  - Tikhonov regularization: eigenvectors of

$$\left(\Sigma^2+\lambda Id\right)^{-1}\Sigma\,\Gamma$$

Regularization (2)

Usual SIR

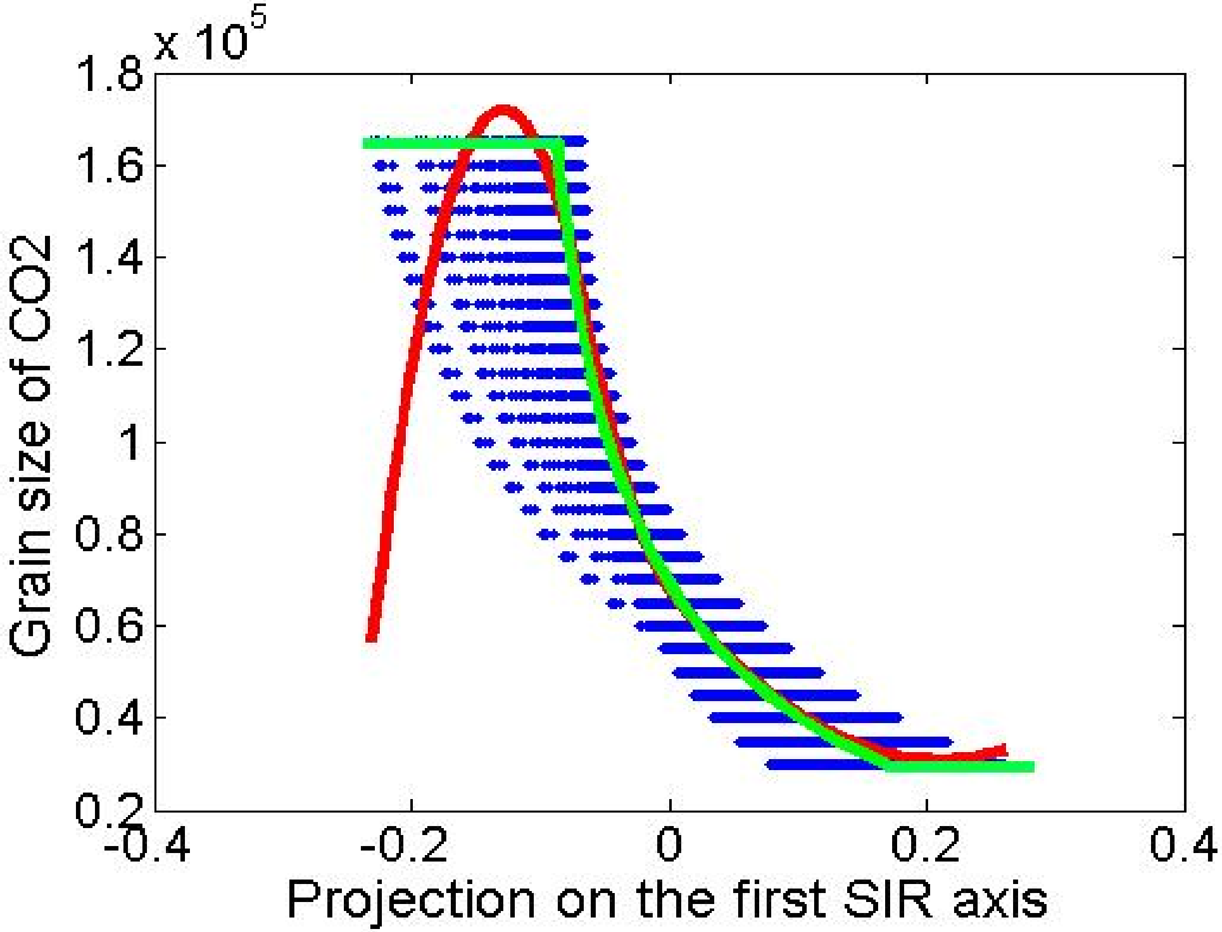
Regularized SIR (Tikhonov)



- *Depends on the regularization parameter λ*
- *The condition number of the matrix decreases when λ increases*
- *The estimation bias increases when λ increases*

Estimation

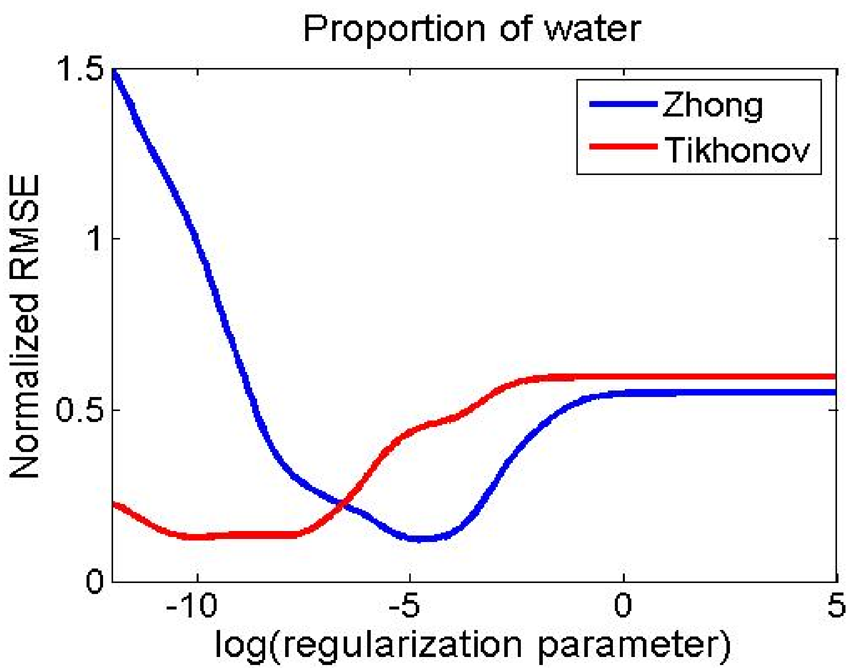
- Nearest neighbors (quite long!)
- Spline functions (choice of new parameters, boundaries)
- **Linear interpolation**

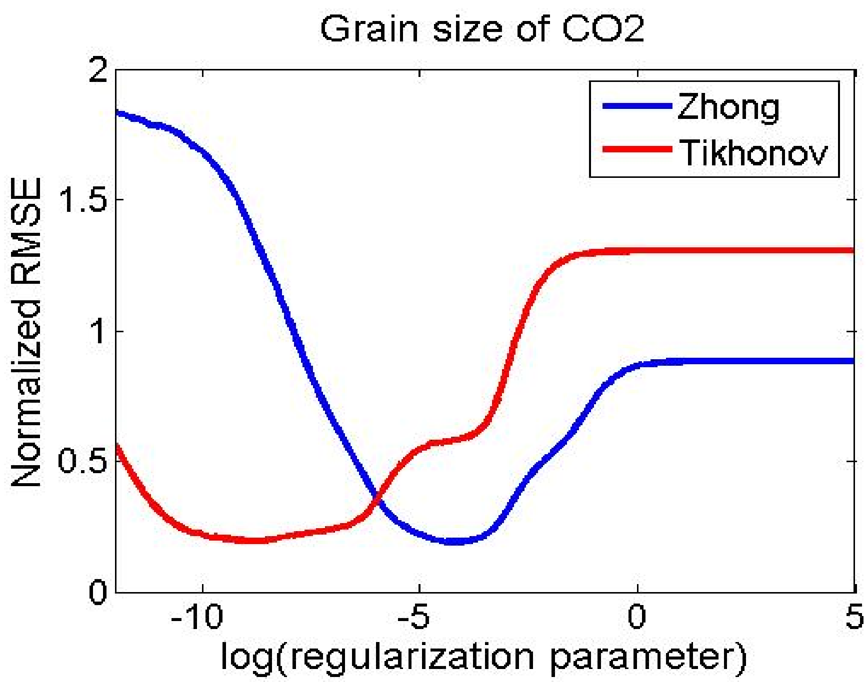


Choice of the regularization parameter

- By minimization of "Normalized RMSE" criterion

$$\frac{\|\hat{y}-y\|}{\|y-\bar{y}\|}=\frac{\text{Residuals sum of square}}{\text{Total sum of square}}$$





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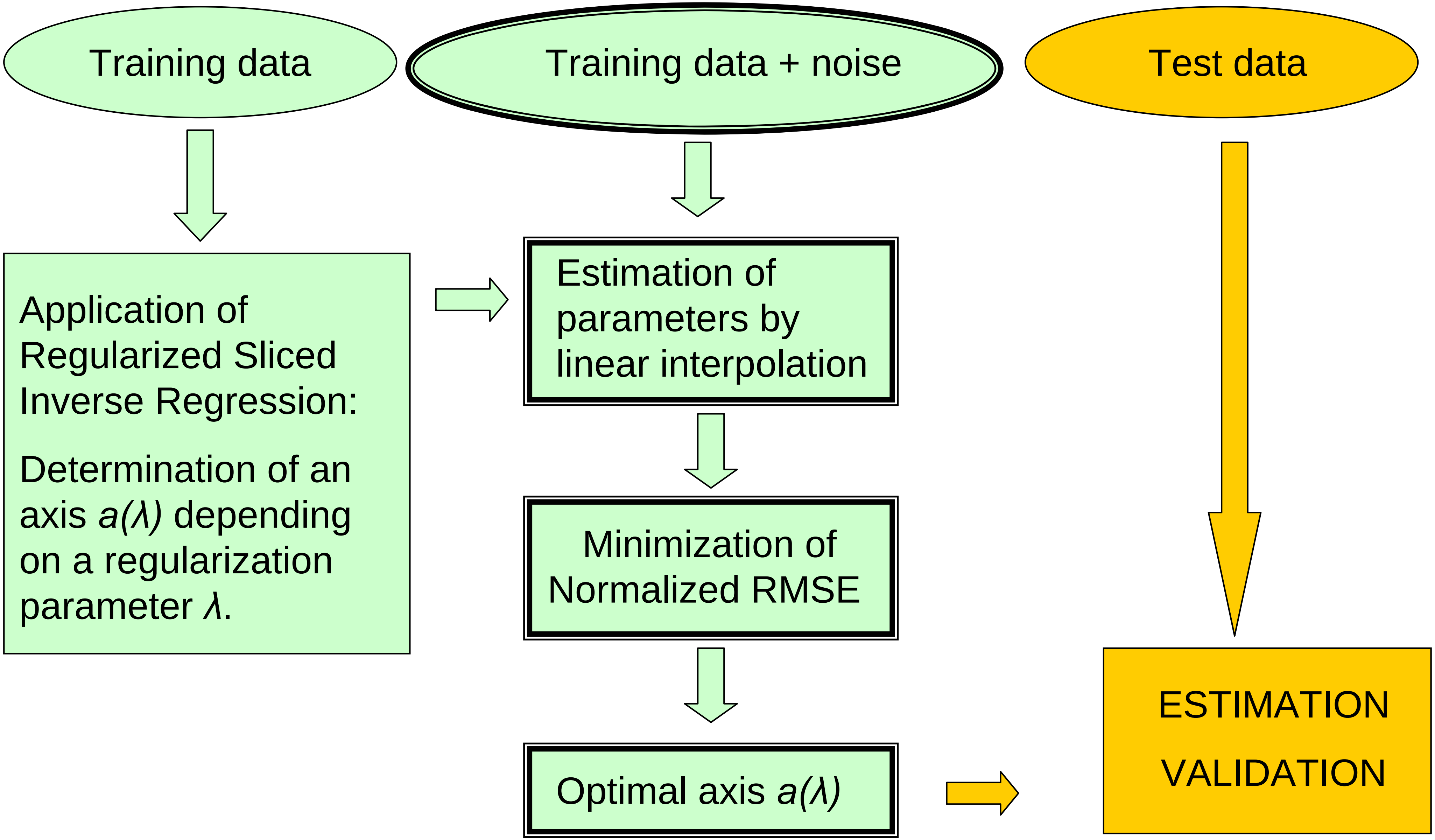
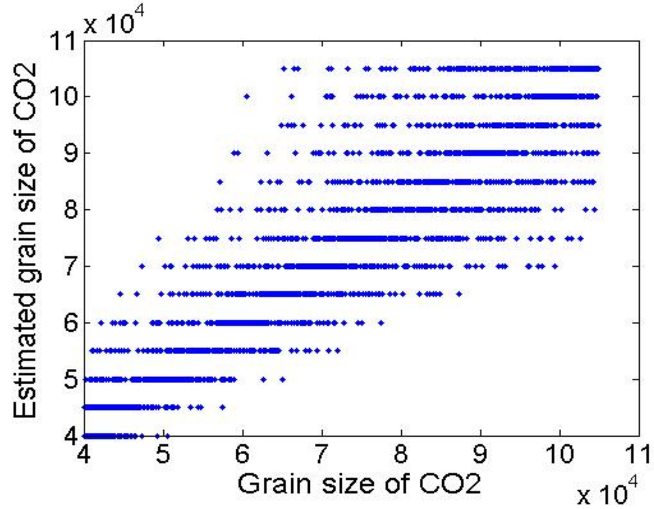
Validation (1)

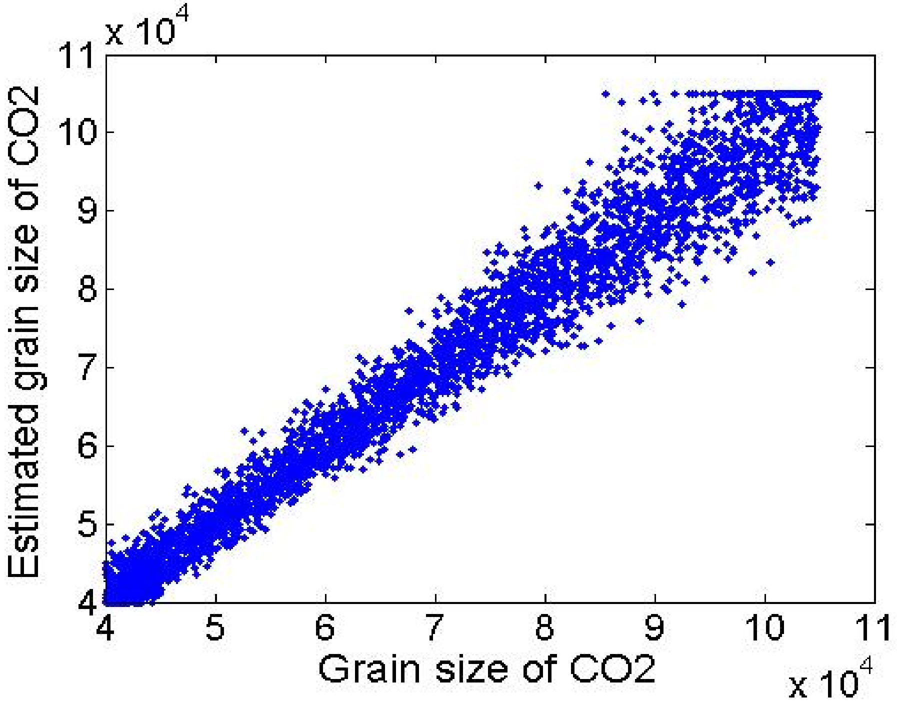


Validation (2)

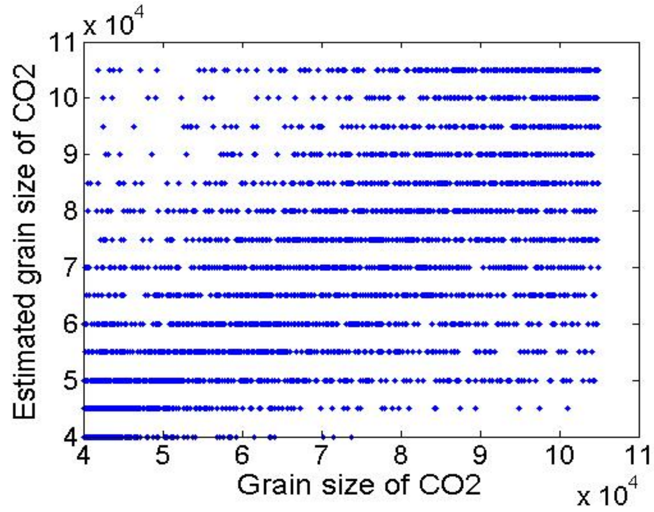
Nearest neighbors



Regularized Sliced inverse regression (Tikhonov)



Weighted nearest neighbors



Validation (3)

Normalized RMSE criterion:

$$\frac{\|\hat{y}-y\|}{\|y-\bar{y}\|}=\frac{\text{Residuals sum of square}}{\text{Total sum of square}}$$

Better when close to 0

SIR criterion:

$$\frac{{}^t a\,\Gamma\,a}{{}^t a\,\Sigma\,a}=\frac{\text{between-slice variance}}{\text{total variance}}$$

Better when close to 1

Parameter	Variation interval	Tikhonov		Zhong		Nearest neighbor	Weighted nearest neighbor
		$\frac{\|\hat{y}-y\|}{\|y-\bar{y}\|}$	$\frac{{}^t a\,\Gamma\,a}{{}^t a\,\Sigma\,a}$	$\frac{\|\hat{y}-y\|}{\|y-\bar{y}\|}$	$\frac{{}^t a\,\Gamma\,a}{{}^t a\,\Sigma\,a}$	$\frac{\|\hat{y}-y\|}{\|y-\bar{y}\|}$	$\frac{\|\hat{y}-y\|}{\|y-\bar{y}\|}$
Proportion of dust	[0.0006 0.002]	0.33	0.92	0.33	0.92	0.56	0.53
Proportion of CO2	[0.9960 0.9988]	0.27	0.96	0.23	0.94	0.56	0.54
Proportion of water	[0.0006 0.002]	0.13	1.00	0.12	1.00	0.27	0.28
Grain size of water	[100 400]	0.37	0.92	0.38	0.87	0.40	0.68
Grain size of CO2	[40000 105000]	0.19	0.99	0.18	0.98	0.38	0.82

- SIR gives better results than nearest neighbor classification
- Tikhonov and Zhong regularizations are equivalent
- With Tikhonov regularization, minimal normalized RMSE is reached on a larger interval than with Zhong's.

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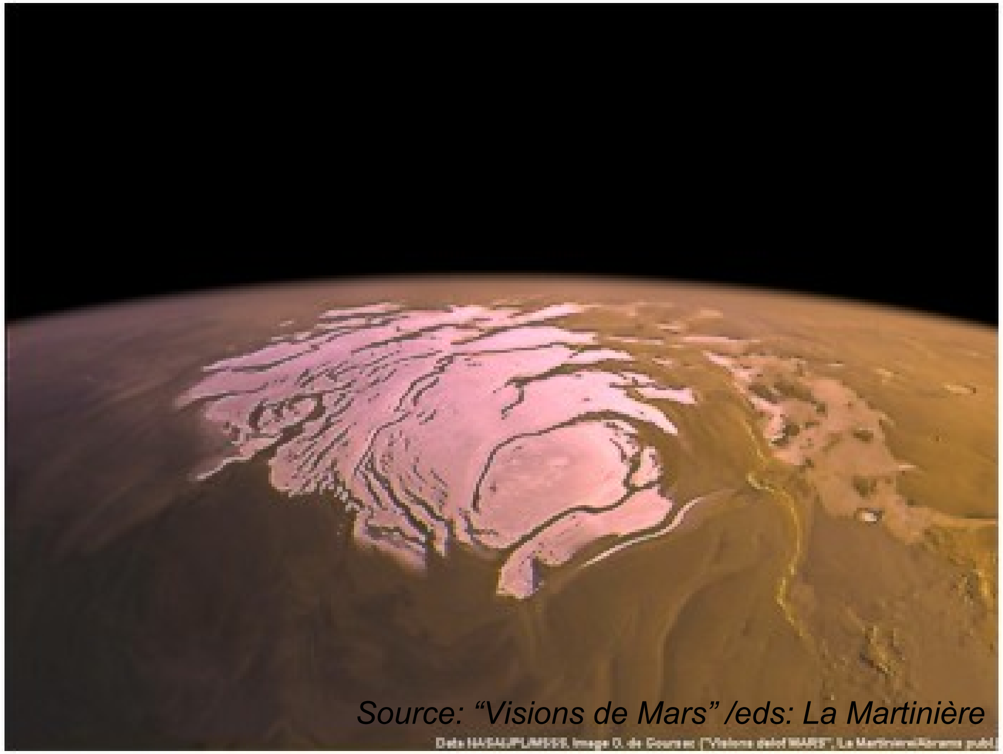
Application to south polar cap of Mars

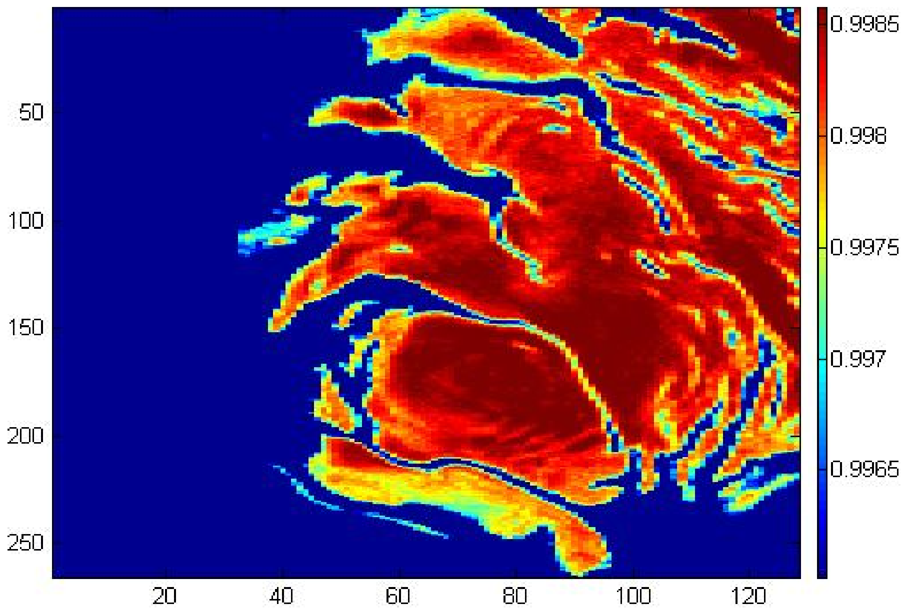


Source: “Visions de Mars” /eds: La Martinière

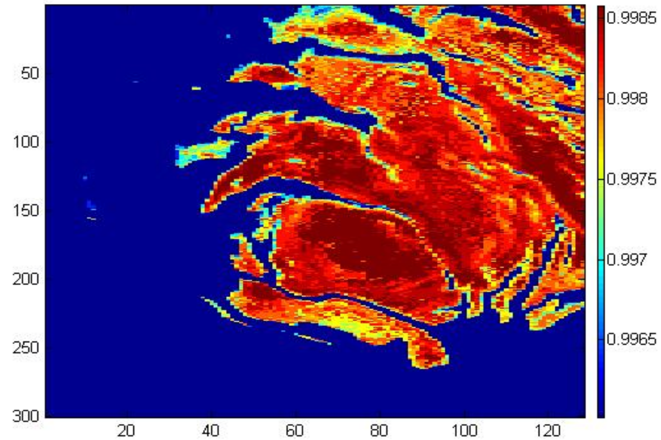
- Model determined by physicists (water + CO_2 + dust)
- 17753 spectra
- 184 wavelengths
- Training data simulated by radiative transfer model
- 5 parameters to study : proportions of water, dust and CO_2, grain sizes of CO_2 and water.

Proportion of CO2

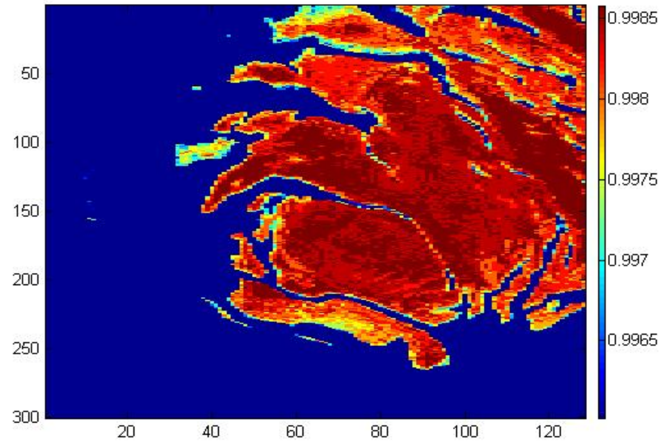
Regularized Sliced Inverse Regression (Tikhonov)



Nearest neighbors

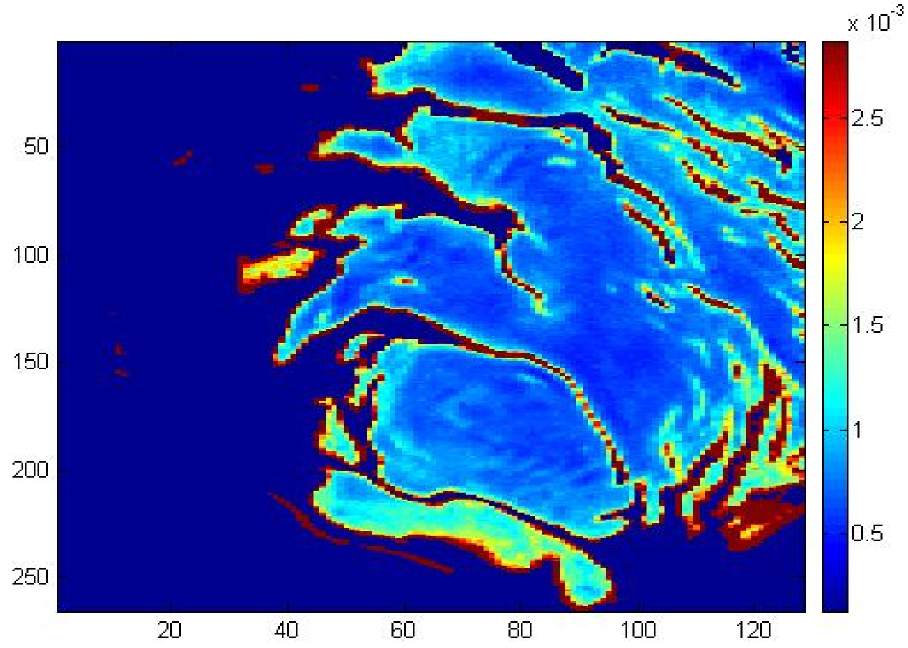


Weighted nearest neighbors

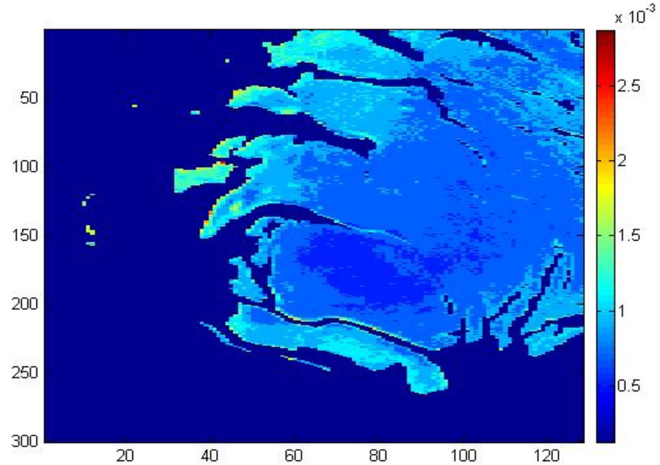


Proportion of water

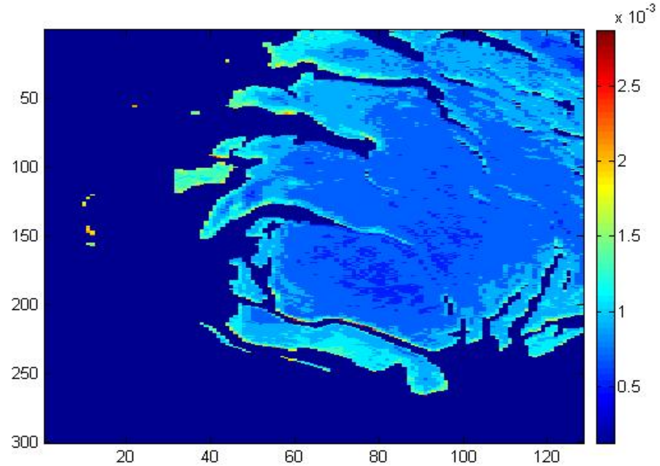
Regularized Sliced Inverse Regression (Tikhonov)



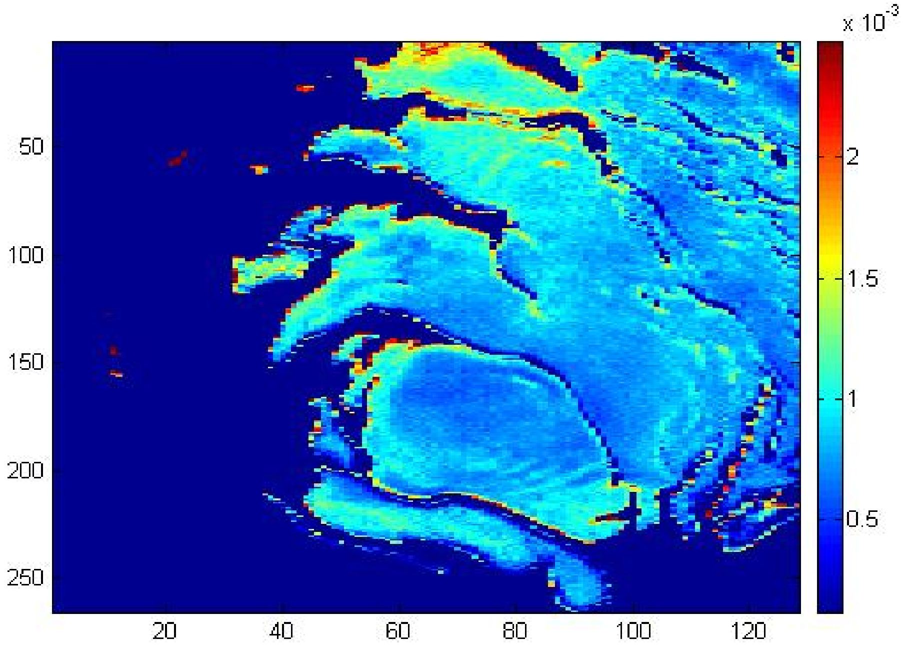
Nearest neighbors

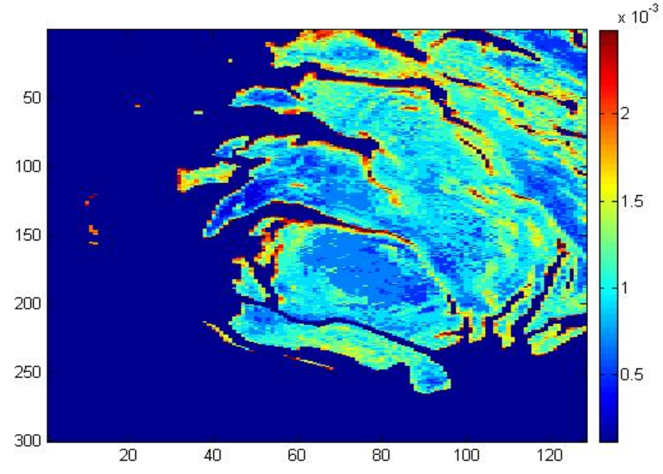


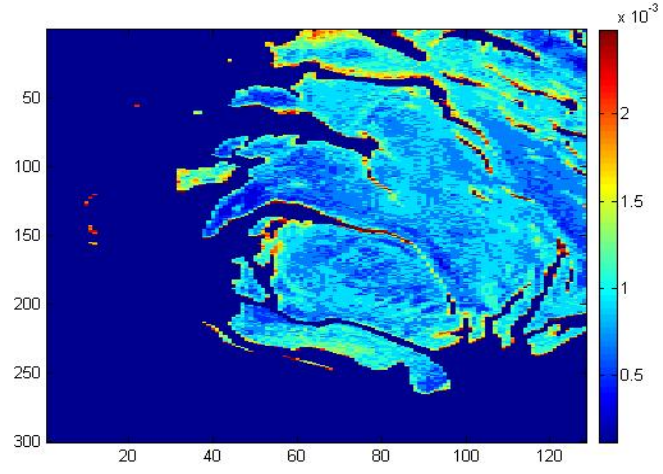
Weighted nearest neighbors



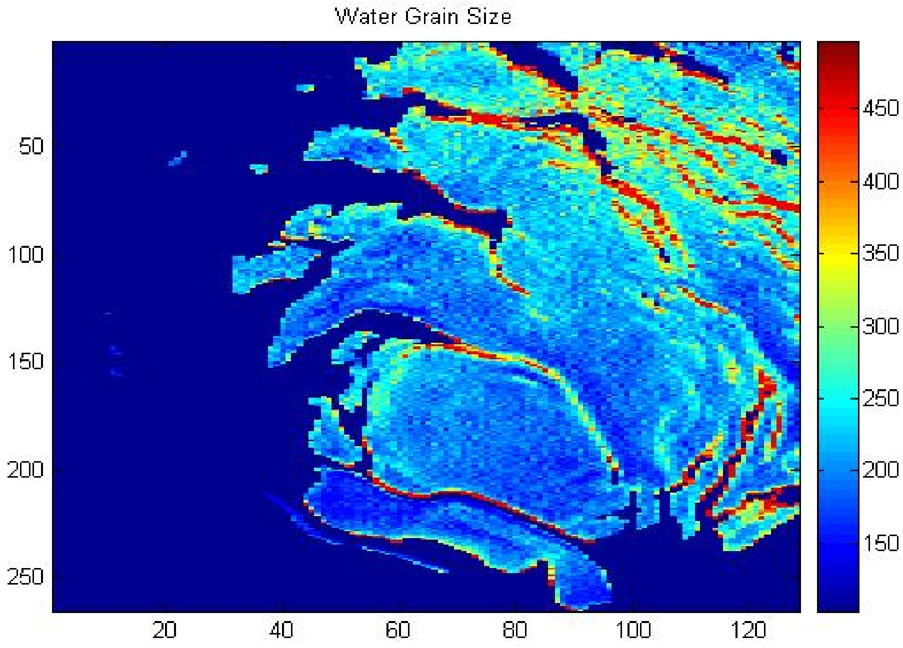
Proportion of Dust

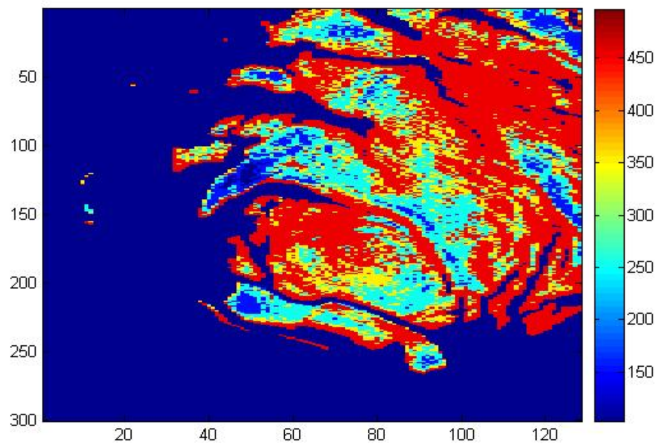


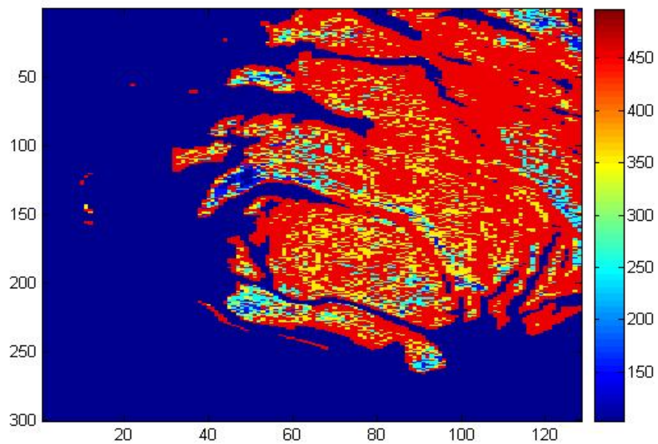




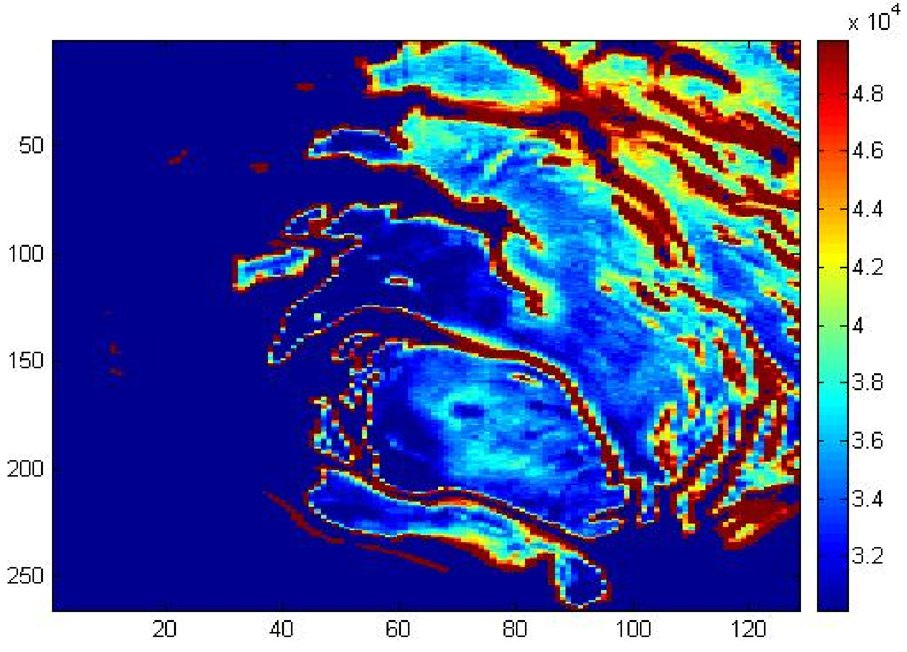
Grain size of water

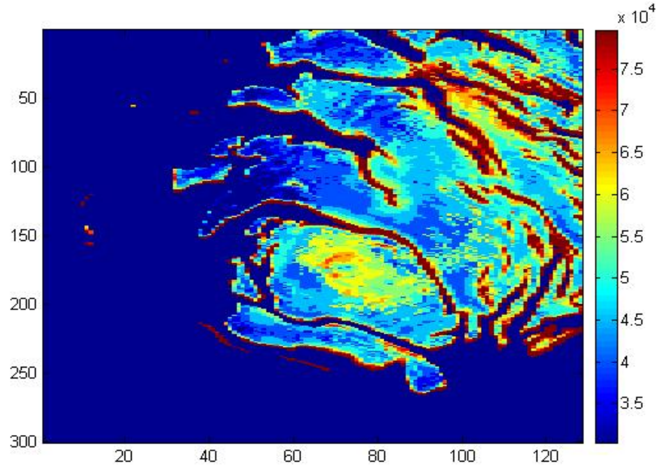


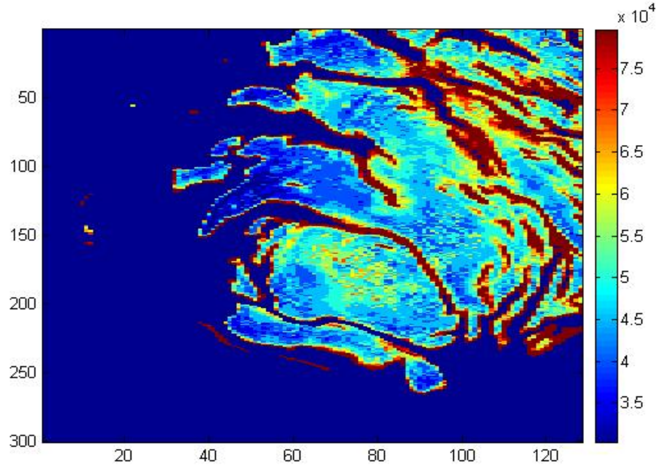




Grain size of CO2







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Conclusion and future work

- Good results on simulations
- Realistic results on real data
- Validation is difficult because of the lack of ground measurements
- Choice of the regularization parameter?
- Uncertainties?
- Comparisons to other methods