Contributions to the Chair "Stress Test"

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1 Ongoing works

1.1 Estimation of the tail index of a mixture of Pareto distributions (with E. Gobet)

We focus on the situation where the data are drawn from a continuous mixture a Pareto-type distributions with tail-index $\gamma \in [\gamma_1, \gamma_2]$ where $\gamma_1 > 0$. We show that the mixture is still heavy-tailed with tail-index γ_2 and that, in this case, the Hill estimator of the tail-index is strongly biased. A bias correction is proposed. Our current work consists in investigating the behavior of this new bias-corrected estimator.

1.2 Importance sampling and extreme-value theory (with C. Albert and E. Gobet)

We investigate the use of extreme-value based proposals in an importance sampling procedure. From a theoretical point of view, we have shown that the use of a generalized Pareto distribution (GPD) leads to super strongly efficient estimators. In practice, the parameters of the GPD have to be estimated and the numerical behavior of the associated procedure has to be evaluated.

1.3 Principal component analysis for extremes (with E. Gobet and A. Sabourin)

We consider a model for multivariate extremes where the tail-index depends on the polar angle. Our aim is to estimate the direction in which the tail-index is the largest, *i.e.* in which the tail is the heaviest. Two approaches can be considered: Either using a nonparametric mode estimator adapted to this context or a stochastic particle method.

2 Results

2.1 Publications

- Tail expectile process and risk assessment `daouia:girard:2019`

  Expectiles define a least squares analogue of quantiles. They are determined by tail expectations rather than tail probabilities. For this reason and many other theoretical and practical merits, expectiles have recently received a lot of attention, especially in actuarial and financial risk management. Their estimation, however, typically requires to consider non-explicit asymmetric least squares estimates rather than the traditional order statistics used for quantile estimation. This makes the study of the tail expectile process a lot harder than that

of the standard tail quantile process. Under the challenging model of heavy-tailed distributions, we derive joint weighted Gaussian approximations of the tail empirical expectile and quantile processes. We then use this powerful result to introduce and study new estimators of extreme expectiles and the standard quantile-based expected shortfall, as well as a novel expectile-based form of expected shortfall.

- Beyond tail median and conditional tail expectation: extreme risk estimation using tail L_p-optimisation `gardes:girard:2019`

  The Conditional Tail Expectation is an indicator of tail behaviour that takes into account both the frequency and magnitude of a tail event. However, the asymptotic normality of its empirical estimator requires that the underlying distribution possess a finite variance; this can be a strong restriction in actuarial and financial applications. A valuable alternative is the Median Shortfall, although it only gives information about the frequency of a tail event. We construct a class of tail L_p- medians encompassing the Median Shortfall and Conditional Tail Expectation. For $p \in (1, 2)$, a tail L_p- median depends on both the frequency and magnitude of tail events, and its empirical estimator is, within the range of the data, asymptotically normal under a condition weaker than a finite variance. We extrapolate this estimator and another technique to extreme levels using the heavy-tailed framework.

- Dependence properties and Bayesian inference for asymmetric multivariate copulas `arbel:crispino:2019`

  We study a broad class of asymmetric copulas introduced by Liebscher as a combination of multiple - usually symmetric-copulas. The main thrust of the paper is to provide new theoretical properties including exact tail dependence expressions and stability properties. A subclass of Liebscher copulas obtained by combining comonotonic copulas is studied in more details. We establish further dependence properties for copulas of this class and show that they are characterized by an arbitrary number of singular components. Furthermore, we introduce a novel iterative representation for general Liebscher copulas which de facto insures uniform margins, thus relaxing a constraint of Liebscher's original construction. Besides, we show that this iterative construction proves useful for inference by developing an Approximate Bayesian computation sampling scheme. This inferential procedure is demonstrated on simulated data and is compared to a likelihood-based approach in a setting where the latter is available.

2.2 Submitted manuscripts

- Nonparametric extreme conditional expectile estimation `girard:stupfler:2019a`

  Expectiles and quantiles can both be defined as the solution of minimization problems. Contrary to quantiles though, expectiles are determined by tail expectations rather than tail probabilities, and define a coherent risk measure. For these two reasons in particular, expectiles have recently started to be considered as serious candidates to become standard tools in actuarial and financial risk management. However, expectiles and their sample versions do not benefit from a simple explicit form, making their analysis significantly harder than that of quantiles and order statistics. This difficulty is compounded when one wishes to integrate auxiliary information about the phenomenon of interest through a finite-dimensional covariate, in which case the problem becomes the estimation of conditional expectiles. In this paper, we exploit the fact that the expectiles of a distribution F are in fact the quantiles of another distribution E explicitly linked to F, in order to construct nonparametric kernel estimators of extreme conditional expectiles.

- Asymptotic behavior of the extrapolation error associated with the estimation of extreme quantiles `albert:dutfoy:2019`

We investigate the asymptotic behavior of the (relative) extrapolation error associated with some estimators of extreme quantiles based on extreme-value theory. It is shown that the extrapolation error can be interpreted as the remainder of a first order Taylor expansion. Conditions are then provided such that this error tends to zero as the sample size increases. Interestingly, in case of the so-called Exponential Tail estimator, these conditions lead to a subdivision of Gumbel maximum domain of attraction into three subsets. In contrast, the extrapolation error associated with Weissman estimator has a common behavior over the whole Fréchet maximum domain of attraction. First order equivalents of the extrapolation error are then derived showing that Weissman estimator may lead to smaller extrapolation errors than the Exponential Tail estimator on some subsets of Gumbel maximum domain of attraction.

- Estimation of the tail-index in a conditional location-scale family of heavy-tailed distributions `ahmad:deme:2019`

  We introduce a location-scale model for conditional heavy-tailed distributions when the covariate is deterministic. First, nonparametric estimators of the location and scale functions are introduced. Second, an estimator of the conditional extreme-value index is derived. The asymptotic properties of the estimators are established under mild assumptions and their finite sample properties are illustrated on simulated data.

2.3 Conferences

`girard:stupfler:2019b,girard:stupfler:2019c`

3 PhD theses

Two PhD theses are connected to the Chair topics:

- Meryem Bousebata (co-advised by Geoffroy Enjolras, CERAG, Université Grenoble Alpes) "*Bayesian estimation of extreme risk measures: Implication for the insurance of natural disasters*", started on October 2018.
- Aboubacrène Ag Ahmad (co-advised by Aliou Diop, Université Gaston-Berger, Sénégal) "*A new location-scale model for heavy-tailed distributions*", started on October 2016.

4 Editorial activities

- Associate Editor of *Statistics and Computing* since 2012.
- Associate Editor of *Journal of Multivariate Analysis* since 2016.
- Associate Editor of *Revstat* since 2019.
- Member of the Advisory Board of *Dependence Modeling* since 2015.

`ahmad:deme:2019` .....
A. Ag Ahmad, E. Deme, A. Diop, and S. Girard.
Estimation of the tail-index in a conditional location-scale family of heavy-tailed distributions.
Submitted, `https://hal.inria.fr/hal-02132976`, 2019.

albert:dutfoy:2019 .....
C. Albert, A. Dutfoy, and S. Girard.
Asymptotic behavior of the extrapolation error associated with the estimation of extreme quantiles.
Submitted, https://hal.inria.fr/hal-01692544, 2019.

arbel:crispino:2019 .....
J. Arbel, M. Crispino, and S. Girard.
Dependence properties and Bayesian inference for asymmetric multivariate copulas.
Journal of Multivariate Analysis, 2019.
To appear.

daouia:girard:2019 .....
A. Daouia, S. Girard, and G. Stupfler.
Tail expectile process and risk assessment.
Bernoulli, 2019.
To appear.

gardes:girard:2019 .....
L. Gardes, S. Girard, and G. Stupfler.
Beyond tail median and conditional tail expectation: extreme risk estimation using tail L_p-optimisation.
Scandinavian Journal of Statistics, 2019.
To appear.

girard:stupfler:2019c .....
S. Girard and G. Stupfler.
Estimation of high-dimensional extreme conditional expectiles.
In *2nd workshop on Multivariate Data and Software*, Limassol, Cyprus, april 2019.

girard:stupfler:2019b .....
S. Girard, G. Stupfler, and A. Usseglio-Carleve.
Nonparametric extreme conditional expectile estimation.
In *11th International Conference on Extreme Value Analysis*, Zagreb, Croatia, june 2019.

girard:stupfler:2019a .....
S. Girard, G. Stupfler, and A. Usseglio-Carleve.
Nonparametric extreme conditional expectile estimation.
Submitted, http://hal.archives-ouvertes.fr/hal-02114255, 2019.