Contributions to the Chair "Stress Test"

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1 Ongoing works

1.1 Estimation of the tail index of a mixture of Pareto distributions (with E. Gobet)

We focus on the situation where the data are drawn from a continuous mixture a Pareto-type distributions with tail-index $\gamma \in [\gamma_1, \gamma_2]$ where $\gamma_1 > 0$. We show that the mixture is still heavy-tailed with tail-index γ_2 and that, in this case, the Hill estimator of the tail-index is strongly biased. A bias correction is introduced thanks to a sharp expansion of the survival distribution of the mixture in the tail. Our current work consists in investigating the behavior of this new bias-corrected estimator (i) from the asymptotic point of view, (ii) on simulated data, and (iii) on real data from environmental measurements.

1.2 Neural networks and extreme-value theory (with M. Allouche and E. Gobet)

The main objective of the PhD thesis of M. Allouche is to build a generative neural networks model based on extreme synthetic data simulated from effective but computationally expensive Monte-Carlo methods. More specifically, the goal is to approximate a quantile $q(u)$ for all level $u \in [0, 1)$ although extreme quantiles (observed as $u \to 1$) are our main region of interest. We moreover focus on extreme quantiles from heavy-tailed distributions *i.e.* belonging to the Fréchet maximum domain of attraction. In such a case, $q(u) \to \infty$ at a power rate as $u \to 1$ and thus, approximating such a function with linear combinations of relu functions is a challenging task for a neural network. To overcome this problem, a nonlinear transformation of the quantile function to another tail function is proposed. The approximation rate of this new function by the neural network is investigated basing on tools from Fourier analysis.

1.3 Principal component analysis for extremes (with E. Gobet and A. Sabourin)

We consider a model for multivariate extremes where the tail-index depends on the polar angle. Our aim is to estimate the direction in which the tail-index is the largest, *i.e.* in which the tail is the heaviest. Two approaches can be considered: Either using a nonparametric mode estimator adapted to this context or a stochastic particle method.

2 Results

2.1 Publications

- Tail expectile process and risk assessment `daouia:girard:2020b`

  Expectiles define a least squares analogue of quantiles. They are determined by tail expectations rather than tail probabilities. For this reason and many other theoretical and practical merits, expectiles have recently received a lot of attention, especially in actuarial and financial risk management. Their estimation, however, typically requires to consider non-explicit asymmetric least squares estimates rather than the traditional order statistics used for quantile estimation. This makes the study of the tail expectile process a lot harder than that of the standard tail quantile process. Under the challenging model of heavy-tailed distributions, we derive joint weighted Gaussian approximations of the tail empirical expectile and quantile processes. We then use this powerful result to introduce and study new estimators of extreme expectiles and the standard quantile-based expected shortfall, as well as a novel expectile-based form of expected shortfall.

- Beyond tail median and conditional tail expectation: extreme risk estimation using tail L_p-optimisation `gardes:girard:2020`

  The Conditional Tail Expectation is an indicator of tail behaviour that takes into account both the frequency and magnitude of a tail event. However, the asymptotic normality of its empirical estimator requires that the underlying distribution possess a finite variance; this can be a strong restriction in actuarial and financial applications. A valuable alternative is the Median Shortfall, although it only gives information about the frequency of a tail event. We construct a class of tail L_p- medians encompassing the Median Shortfall and Conditional Tail Expectation. For $p \in (1, 2)$, a tail L_p- median depends on both the frequency and magnitude of tail events, and its empirical estimator is, within the range of the data, asymptotically normal under a condition weaker than a finite variance. We extrapolate this estimator and another technique to extreme levels using the heavy-tailed framework.

- ExpectHill estimation, extreme risk and heavy tails `daouia:girard:2020`

  Risk measures of a financial position are, from an empirical point of view, mainly based on quantiles. Replacing quantiles with their least squares analogues, called expectiles, has recently received increasing attention. The novel expectile-based risk measures satisfy all coherence requirements. We revisit their extreme value estimation for heavy-tailed distributions. First, we estimate the underlying tail index via weighted combinations of top order statistics and asymmetric least squares estimates. The resulting expectHill estimators are then used as the basis for estimating tail expectiles and Expected Shortfall. The asymptotic theory of the proposed estimators is provided, along with numerical simulations and applications to actuarial and financial data.

- Asymptotic behavior of the extrapolation error associated with the estimation of extreme quantiles `albert:dutfoy:2020`

  We investigate the asymptotic behavior of the (relative) extrapolation error associated with some estimators of extreme quantiles based on extreme-value theory. It is shown that the extrapolation error can be interpreted as the remainder of a first order Taylor expansion. Conditions are then provided such that this error tends to zero as the sample size increases. Interestingly, in case of the so-called Exponential Tail estimator, these conditions lead to a subdivision of Gumbel maximum domain of attraction into three subsets. In contrast, the extrapolation error associated with Weissman estimator has a common behavior over the whole Fréchet maximum domain of attraction. First order equivalents of the extrapolation error are then derived showing that Weissman estimator may lead to smaller extrapolation

errors than the Exponential Tail estimator on some subsets of Gumbel maximum domain of attraction.

2.2 Submitted manuscripts

- Nonparametric extreme conditional expectile estimation `girard:stupfler:2020a`

  Expectiles and quantiles can both be defined as the solution of minimization problems. Contrary to quantiles though, expectiles are determined by tail expectations rather than tail probabilities, and define a coherent risk measure. For these two reasons in particular, expectiles have recently started to be considered as serious candidates to become standard tools in actuarial and financial risk management. However, expectiles and their sample versions do not benefit from a simple explicit form, making their analysis significantly harder than that of quantiles and order statistics. This difficulty is compounded when one wishes to integrate auxiliary information about the phenomenon of interest through a finite-dimensional covariate, in which case the problem becomes the estimation of conditional expectiles. In this paper, we exploit the fact that the expectiles of a distribution F are in fact the quantiles of another distribution E explicitly linked to F, in order to construct nonparametric kernel estimators of extreme conditional expectiles.

- On the estimation of the variability in the distribution tail `gardes:girard:2020b`

  We propose a new measure of variability in the tail of a distribution by applying a Box-Cox transformation of parameter $p \geq 0$ to the tail-Gini functional. It is shown that the so-called Box-Cox Tail Gini Variability measure is a valid variability measure whose condition of existence may be as weak as necessary thanks to the tuning parameter p. The tail behaviour of the measure is investigated under a general extreme-value condition on the distribution tail. We then show how to estimate the Box-Cox Tail Gini Variability measure within the range of the data. These methods provide us with basic estimators that are then extrapolated using the extreme-value assumption to estimate the variability in the very far tails. The finite sample behavior of the estimators is illustrated both on simulated and real data.

- Estimation of extreme quantiles from heavy-tailed distributions in a location-dispersion regression models `ahmad:deme:2020`

  We consider a location-dispersion regression model for heavy-tailed distributions when the multidimensional covariate is deterministic. In a first step, nonparametric estimators of the regression and dispersion functions are introduced. This permits, in a second step, to derive an estimator of the conditional extreme-value index computed on the residuals. Finally, a plug-in estimator of extreme conditional quantiles is built using these two preliminary steps. It is shown that the resulting semi-parametric estimator is asymptotically Gaussian and may benefit from the same rate of convergence as in the unconditional situation. Its finite sample properties are illustrated both on simulated and real tsunami data.

- Extreme conditional expectile estimation in heavy-tailed heteroscedastic regression models `girard:stupfler:2020`

  Expectiles define a least squares analogue of quantiles. They have been the focus of a substantial quantity of research in the context of actuarial and financial risk assessment over the last 10 years. Unlike quantiles, expectiles induce coherent risk measures and are calculated using tail expectations rather than merely tail probabilities ; contrary to the popular quantile-based Expected Shortfall, they define elicitable risk measures. The behaviour and estimation of extreme expectiles using independent and identically distributed heavy-tailed observations has been investigated in a recent series of papers. The case of extreme conditional expectile estimation has, however, not been addressed so far in the literature. We build here a general theory for the estimation of extreme conditional expectiles in heteroscedastic

regression models with heavy-tailed noise; an important feature of our approach is that it is intended to cope with covariates having a large but fixed dimension. We demonstrate how our results can be applied to a wide class of important examples, among which linear heteroscedastic models, heteroscedastic single-index models and autoregressive time series models. Our estimators are showcased on a numerical simulation study, as well as on real sets of actuarial and financial data.

2.3 Conferences

Nothing to report.

3 PhD theses supervision

One PhD thesis is founded by the Chair:

- Michael Allouche (co-advised with Emmanuel Gobet, Ecole Polytechnique) "*Simulation of extreme events with IA generative models and applications to risk management in banking"*, started on April 2020.

Two PhD theses are connected to the Chair topics:

- Meryem Bousebata (co-advised with Geoffroy Enjolras, CERAG, Université Grenoble Alpes) "*Bayesian estimation of extreme risk measures: Implication for the insurance of natural disasters*", started on October 2018.
- Aboubacrène Ag Ahmad (co-advised with Aliou Diop, Université Gaston-Berger, Sénégal) "*A new location-scale model for heavy-tailed distributions*", should be defended on September 2020.

4 Editorial activities

- Associate Editor of *Statistics and Computing* since 2012.
- Associate Editor of *Journal of Multivariate Analysis* since 2016.
- Associate Editor of *Revstat* since 2019.
- Member of the Advisory Board of *Dependence Modeling* since 2015.

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