Contributions to the Chair "Stress Test"

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1 Ongoing works

1.1 Estimation of extreme quantile from a mixture of Pareto distributions (with E. Gobet)

We focus on the situation where the data are drawn from a continuous mixture a Pareto-type distributions with tail-index $\gamma \in [\gamma_1, \gamma_2]$ where $\gamma_1 > 0$. We show that the mixture is still heavy-tailed with tail-index γ_2 and that, in this case, the Hill estimator of the tail-index is strongly biased. A bias correction is introduced thanks to a sharp expansion of the survival distribution of the mixture in the tail. An estimator of extreme quantiles is then derived. Our current work consists in investigating the behavior of this new bias-corrected estimator (i) from the asymptotic point of view, (ii) on simulated data, and (iii) on financial data.

1.2 Estimation of extreme quantiles from heavy-tailed distributions with neural networks (with M. Allouche and E. Gobet)

In the context of the PhD thesis of M. Allouche, we propose a new parametrization for one-hidden layer eLU neural networks able to estimate extreme quantiles, starting from heavy-tailed data. We provide an analysis of the uniform error between the extreme log-quantile and its neural network approximation. Numerical experiments are conducted on simulated data to compare the performance of our method with other estimators from the literature. The method is then extended to the estimation of conditional extreme quantiles by combining multi-layers ReLU neural networks with the previous one layer eLU neural network. An illustration is provided on rainfall data in the Cévennes-Vivarais region.

2 Results

2.1 Publications

- Generative model for fBm with deep ReLU neural networks `allouche:girard:2022`

  We provide a large probability bound on the uniform approximation of fractional Brownian motion $(B^H(t), t \in [0, 1])$ with Hurst parameter H, by a deep-feedforward ReLU neural network fed with a N-dimensional Gaussian vector, with bounds on the network construction (number of hidden layers and total number of neurons). Essentially, up to log terms, achieving an uniform error of $\mathcal{O}(N^{-H})$ is possible with $\log(N)$ hidden layers and $\mathcal{O}(N \log N)$ parameters. Our analysis relies, in the standard Brownian motion case ($H = 1/2$), on the Levy construction of B^H and in the general fractional Brownian motion case ($H \neq 1/2$), on the Lemarié-Meyer wavelet representation of B^H. This work gives theoretical support on new generative models based on neural networks for simulating continuous-time processes.

- EV-GAN: Simulation of extreme events with ReLU neural networks `allouche:girard:2022bis`

  Feedforward neural networks based on Rectified linear units (ReLU) cannot efficiently approximate quantile functions which are not bounded, especially in the case of heavy-tailed distributions. We thus propose a new parametrization for the generator of a Generative adversarial network (GAN) adapted to this framework, basing on extreme-value theory. We provide an analysis of the uniform error between the extreme quantile and its GAN approximation. It appears that the rate of convergence of the error is mainly driven by the second-order parameter of the data distribution. The above results are illustrated on simulated data and real financial data.

- Functional estimation of extreme conditional expectiles `girard:stupfler:2022`

  Quantiles and expectiles can be interpreted as solutions of convex minimization problems. Unlike quantiles, expectiles are determined by tail expectations rather than tail probabilities, and define a coherent risk measure. For these reasons, among others, they have recently been the subject of renewed attention in actuarial and financial risk management. Here, we focus on the challenging problem of estimating extreme expectiles, whose order converges to one as the sample size increases, given a functional covariate. We construct a functional kernel estimator of extreme conditional expectiles by writing expectiles as quantiles of a different distribution. The asymptotic properties of the estimators are studied in the context of conditional heavy-tailed distributions. We also provide and analyse different ways of estimating the functional tail index, as a way to extrapolate our estimates to the very far conditional tails. A numerical illustration of the finite-sample performance of our estimators is provided on simulated and real datasets.

- Nonparametric extreme conditional expectile estimation `girard:stupfler:2022bis`

  Expectiles and quantiles can both be defined as the solution of minimization problems. Contrary to quantiles though, expectiles are determined by tail expectations rather than tail probabilities, and define a coherent risk measure. For these two reasons in particular, expectiles have recently started to be considered as serious candidates to become standard tools in actuarial and financial risk management. However, expectiles and their sample versions do not benefit from a simple explicit form, making their analysis significantly harder than that of quantiles and order statistics. This difficulty is compounded when one wishes to integrate auxiliary information about the phenomenon of interest through a finite-dimensional covariate, in which case the problem becomes the estimation of conditional expectiles. In this paper, we exploit the fact that the expectiles of a distribution F are in fact the quantiles of another distribution E explicitly linked to F, in order to construct nonparametric kernel estimators of extreme conditional expectiles.

- On automatic bias reduction for extreme expectile estimation `girard:stupfler:2022ter`

  Expectiles induce a law-invariant risk measure that has recently gained popularity in actuarial and financial risk management applications. Unlike quantiles or the quantile-based Expected Shortfall, the expectile risk measure is coherent and elicitable. The estimation of extreme expectiles in the heavy-tailed framework, which is reasonable for extreme financial or actuarial risk management, is not without difficulties; currently available estimators of extreme expectiles are typically biased and hence may show poor finite-sample performance even in fairly large samples. We focus here on the construction of bias-reduced extreme expectile estimators for heavy-tailed distributions. The rationale for our construction hinges on a careful investigation of the asymptotic proportionality relationship between extreme expectiles and their quantile counterparts, as well as of the extrapolation formula motivated by the heavy-tailed context. We accurately quantify and estimate the bias incurred by the use of these relationships when constructing extreme expectile estimators. This motivates the introduction of a class of bias-reduced estimators whose asymptotic properties are rigorously shown, and whose finite-sample properties are assessed on a simulation study and three samples of real data from economics, insurance and finance.

2.2 Submitted manuscripts

- Extreme Partial Least-Squares regression `bousebata:enjolras:2022`

  We propose a new approach, called Extreme-PLS, for dimension reduction in regression and adapted to distribution tails. The objective is to find linear combinations of predictors that best explain the extreme values of the response variable in a non-linear inverse regression model. The asymptotic normality of the Extreme-PLS estimator is established in the single-index framework and under mild assumptions. The performance of the method is assessed on simulated data. A statistical analysis of French farm income data, considering extreme cereal yields, is provided as an illustration.

- A refined Weissman estimator for extreme quantiles: `allouche:elmethni:2022`

  Weissman extrapolation methodology for estimating extreme quantiles from heavy-tailed distributions is based on two estimators: an order statistic to estimate an intermediate quantile and an estimator of the tail-index. The common practice is to select the same intermediate sequence for both estimators. In this work, we show how an adapted choice of two different intermediate sequences leads to a reduction of the asymptotic bias associated with the resulting refined Weissman estimator. The asymptotic normality of the latter estimator is established and a data-driven method is introduced for the practical selection of the intermediate sequences. Our approach is compared to Weissman estimator and to six bias reduced estimators of extreme quantiles on a large scale simulation study. It appears that the refined Weissman estimator outperforms its competitors in a wide variety of situations, especially in the challenging high bias cases. Finally, an illustration on an actuarial real data set is provided.

2.3 Conferences

- Member of Scientific Program Committee of the 15th International Conference of the ERCIM WG on Computational and Methodological Statistics (CMStatistics 2022), King's College London, 17–19 December 2022. Organizer of an invited session "Machine learning for extremes" (5 invited talks to be confirmed).

- Some of the above works have been presented at international conferences as invited talks: `elmethni:girard:2021bis,girard:gobet:2021,bousebata:enjolras:2021bis,usseglio:girard:2021bis` or in contributed sessions: `allouche:girard:2021bis,usseglio:girard:2021,bousebata:enjolras:2021`, `allouche:girard:2021,elmethni:girard:2021`.

3 PhD theses supervision

One PhD thesis is founded by the Chair:

- Michael Allouche (co-advised with Emmanuel Gobet, Ecole Polytechnique) "*Simulation of extreme events with IA generative models and applications to risk management in banking*", started on April 2020.

One PhD thesis is connected to the Chair topics:

- Meryem Bousebata (co-advised with Geoffroy Enjolras, CERAG, Université Grenoble Alpes) "*Bayesian estimation of extreme risk measures: Implication for the insurance of natural disasters*", defended on March 2022.

4 Editorial activities

- Associate Editor of *Journal of Multivariate Analysis* since 2016.
- Associate Editor of *Revstat* since 2019.
- Member of the Advisory Board of *Dependence Modeling* since 2015.

`allouche:elmethni:2022` ....
M. Allouche, J. El-methni, and S. Girard.
A refined Weissman estimator for extreme quantiles.
`https://hal.inria.fr/hal-03266676`, 2022.

`allouche:girard:2021bis` ....
M. Allouche, S. Girard, and E. Gobet.
Generative model for fbm with deep ReLU neural networks.
In *Bernoulli-IMS 10th World Congress in Probability and Statistics*, Seoul, South Korea / Virtual, july 2021.

`allouche:girard:2021` ....
M. Allouche, S. Girard, and E. Gobet.
On the approximation of extreme quantiles with ReLU neural networks.
In *12th International Conference on Extreme Value Analysis*, Edinburgh, UK / Virtual, june 2021.

`allouche:girard:2022bis` ....
M. Allouche, S. Girard, and E. Gobet.
EV-GAN: Simulation of extreme events with ReLU neural networks.
Journal of Machine Learning Research, 2022.
to appear.

`allouche:girard:2022` ....
M. Allouche, S. Girard, and E. Gobet.
Generative model for fBm with deep ReLU neural networks.
Journal of Complexity, 2022.
to appear.

`bousebata:enjolras:2021bis` ....
M. Bousebata, G. Enjolras, and S. Girard.
Extreme partial least-squares regression.
In *14th International Conference of the ERCIM WG on Computational and Methodological Statistics*, London, United Kingdom, 2021.

`bousebata:enjolras:2021` ....
M. Bousebata, G. Enjolras, and S. Girard.
Extreme partial least-squares regression.
In *12th International Conference on Extreme Value Analysis*, Edinburgh, UK / Virtual, june 2021.

`bousebata:enjolras:2022` ....

M. Bousebata, G. Enjolras, and S. Girard.
Extreme Partial Least-Squares.
`https://hal.inria.fr/hal-03165399`, 2022.

`elmethni:girard:2021bis` ....

J. El Methni and S. Girard.
A bias-reduced version of the Weissman estimator for extreme value-at-risk.
In *14th International Conference of the ERCIM WG on Computational and Methodological Statistics*, London, United Kingdom, 2021.

`elmethni:girard:2021` ....

J. El-Methni and S. Girard.
A bias-reduced version of the Weissman extreme quantile estimator.
In *12th International Conference on Extreme Value Analysis*, Edinburgh, UK / Virtual, june 2021.

`girard:gobet:2021` ....

S. Girard and E. Gobet.
Estimation of the largest tail-index and extreme quantiles from a mixture of heavy-tailed distributions.
In *14th International Conference of the ERCIM WG on Computational and Methodological Statistics*, London, United Kingdom, 2021.

`girard:stupfler:2022` ....

S. Girard, G. Stupfler, and A. Usseglio-Carleve.
Functional estimation of extreme conditional expectiles.
Econometrics and Statistics, 49(1):78–115, 2022.

`girard:stupfler:2022bis` ....

S. Girard, G. Stupfler, and A. Usseglio-Carleve.
Nonparametric extreme conditional expectile estimation.
Scandinavian Journal of Statistics, 49(1):78–115, 2022.

`girard:stupfler:2022ter` ....

S. Girard, G. Stupfler, and A. Usseglio-Carleve.
On automatic bias reduction for extreme expectile estimation.
Statistics and Computing, 2022.
to appear.

`usseglio:girard:2021bis` ....

A. Usseglio-Carleve, S. Girard, and G. Stupfler.
Extreme conditional expectile estimation in heavy-tailed heteroscedastic regression models.
In *14th International Conference of the ERCIM WG on Computational and Methodological Statistics*, London, United Kingdom, 2021.

`usseglio:girard:2021` ....

A. Usseglio-Carleve, S. Girard, and G. Stupfler.
Extreme expectile regression: theory and applications.
In *12th International Conference on Extreme Value Analysis*, Edinburgh, UK / Virtual, june 2021.