Research results in 2019

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Abstract

This note summarizes my research results in 2019, mainly focused on extreme-value analysis and copulas.

1 Estimation of extreme risk measures

One of the most popular risk measures is the Value-at-Risk (VaR) introduced in the 1990's. In statistical terms, the VaR at level $\alpha \in (0,1)$ corresponds to the upper α-quantile of the loss distribution. The Value-at-Risk however suffers from several weaknesses. First, it provides us only with a pointwise information: VaR(α) does not take into consideration what the loss will be beyond this quantile. Second, random loss variables with light-tailed distributions or heavy-tailed distributions may have the same Value-at-Risk. Finally, Value-at-Risk is not a coherent risk measure since it is not subadditive in general. A first coherent alternative risk measure is the Conditional Tail Expectation (CTE), also known as Tail-Value-at-Risk, Tail Conditional Expectation or Expected Shortfall in case of a continuous loss distribution. The CTE is defined as the expected loss given that the loss lies above the upper α-quantile of the loss distribution. This risk measure thus takes into account the whole information contained in the upper tail of the distribution.

However, the asymptotic normality of the empirical CTE estimator requires that the underlying distribution possess a finite variance; this can be a strong restriction in heavy-tailed models which constitute the favoured class of models in actuarial and financial applications. One possible solution in very heavy-tailed models where this assumption fails could be to use the more robust Median Shortfall, but this quantity is actually just a quantile, which therefore only gives information about the frequency of a tail event and not about its typical magnitude. In [1], we construct a synthetic class of tail L_p-medians, which encompasses the Median Shortfall (for $p = 1$) and Conditional Tail Expectation (for $p = 2$). We show that, for $1 < p < 2$, a tail L_p-median always takes into account both the frequency and magnitude of tail events, and its empirical estimator is, within the range of the data, asymptotically normal under a condition weaker than a finite variance. We extrapolate this estimator, along with another technique, to proper extreme levels using the heavy-tailed framework. The estimators are showcased on a simulation study and on a set of real fire insurance data showing evidence of a very heavy right tail.

A possible coherent alternative risk measure is based on expectiles [2]. Compared to quantiles, the family of expectiles is based on squared rather than absolute error loss minimization. The flexibility and virtues of these least squares analogues of quantiles are now well established in actuarial science, econometrics and statistical finance. have recently received a lot of attention, especially in actuarial and financial risk man-agement. Their estimation, however, typically requires to consider non-explicit asymmetric leasts-quares estimates rather than the traditional order statistics used for quantile estimation. This makes the study of the tail expectile process a lot harder than that of the standard tail quantile process. Under the challenging model of heavy-tailed distributions, we derive joint weighted Gaussian approximations of the tail empirical expectile and quantile processes. We then use this powerful result to introduce and study new estimators of extreme expectiles and the standard quantile-based expected shortfall, as well as a novel expectile-based form of expected shortfall [3].

Both quantiles and expectiles were embedded in the more general class of L_p-quantiles [4] as the minimizers of a generic asymmetric convex loss function. It has been proved very recently that the only L_p-quantiles that are coherent risk measures are the expectiles. Here, we work in a context of heavy tails, which is especially relevant to actuarial science, finance, econometrics and natural sciences, and we construct an estimator of the tail index of the underlying distribution based on extreme L_p-quantiles. We establish the asymptotic normality of such an estimator and in doing so, we extend very recent results on extreme expectile and L_p-quantile estimation. We provide a discussion of the choice of p in practice, as well as a methodology for reducing the bias of our estimator. Its finite-sample performance is evaluated on simulated data and on a set of real hydrological data.

2 Conditional extremal events

The goal of the PhD thesis of Aboubacrene Ag Ahmad is to contribute to the development of theoretical and algorithmic models to tackle conditional extreme value analysis, ie the situation where some covariate information X is recorded simultaneously with a quantity of interest Y. In such a case, extreme quantiles and expectiles are functions of the covariate. In [5], we consider a location-scale model for conditional heavy-tailed distributions when the covariate is deterministic. First, nonparametric estimators of the location and scale functions are introduced. Second, an estimator of the conditional extreme-value index is derived. The asymptotic properties of the estimators are established under mild assumptions and their finite sample properties are illustrated both on simulated and real data.

Expectiles have recently started to be considered as serious candidates to become standard tools in actuarial and financial risk management. However, expectiles and their sample versions do not benefit from a simple explicit form, making their analysis significantly harder than that of quantiles and order statistics. This difficulty is compounded when one wishes to integrate auxiliary information about the phenomenon of interest through a finite-dimensional covariate, in which case the problem becomes the estimation of conditional expectiles. In [6], we exploit the fact that the expectiles of a distribution F are in fact the quantiles of another distribution E explicitly linked to F, in order to construct nonparametric kernel estimators of extreme conditional expectiles. We analyze the asymptotic properties of our estimators in the context of conditional heavy-tailed distributions. Applications to simulated data and real insurance data are provided. The results are submitted for publication.

3 Estimation of the variability in the distribution tail

We propose a new measure of variability in the tail of a distribution by applying a Box-Cox transformation of parameter $p \geq 0$ to the tail-Gini functional. It is shown that the so-called Box-Cox Tail Gini Variability measure is a valid variability measure whose condition of existence may be as weak as necessary thanks to the tuning parameter p. The tail behaviour of the measure is investigated under a general extreme-value condition on the distribution tail. We then show how to estimate the Box-Cox Tail Gini Variability measure within the range of the data. These methods provide us with basic estimators that are then extrapolated using the extreme-value assumption to estimate the variability in the very far tails. The finite sample behavior of the estimators is illustrated both on simulated and real data. This work is submitted for publication [7].

4 Extrapolation limits of extreme-value methods

The PhD thesis of Clément Albert (co-funded by EDF) is dedicated to the study of the sensitivity of extreme-value methods to small changes in the data and to their extrapolation ability. Two directions are explored:

(i) In [8], we investigate the asymptotic behavior of the (relative) extrapolation error associated with some estimators of extreme quantiles based on extreme-value theory. It is shown that the extrapolation error can be interpreted as the remainder of a first order Taylor expansion. Necessary and sufficient conditions are then provided such that this error tends to zero as the sample size increases. Interestingly, in case of the so-called Exponential Tail estimator, these conditions lead to a subdivision of Gumbel maximum domain of attraction into three subsets. In constrast, the extrapolation error associated with Weissman estimator has a common behavior over the whole Fréchet maximum domain of attraction. First order equivalents of the extrapolation error are then derived and their accuracy is illustrated numerically.

(ii) [9], We propose a new estimator for extreme quantiles under the log-generalized Weibull-tail model, introduced by Cees de Valk. This model relies on a new regular variation condition which, in some situations, permits to extrapolate further into the tails than the classical assumption in extreme-value theory. The asymptotic normality of the estimator is established and its finite sample properties are illustrated both on simulated and real datasets.

5 Bayesian inference for copulas

We study in [10] a broad class of asymmetric copulas known as Liebscher copulas and defined as a combination of multiple – usually symmetric – copulas. The main thrust of this work is to provide new theoretical properties including exact tail dependence expressions and stability properties. A subclass of Liebscher copulas obtained by combining Fréchet copulas is studied in more details. We establish further dependence properties for copulas of this class and show that they are characterized by an arbitrary number of singular components. Furthermore, we introduce a novel iterative construction for general Liebscher copulas which de facto insures uniform margins, thus relaxing a constraint of Liebscher's original construction. Besides, we show that this iterative construction proves useful for inference by developing an Approximate Bayesian computation sampling scheme. This inferential procedure is demonstrated on simulated data.

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