Workshop "Copulas and Extremes"

Organization : Marianne Clausel & Stéphane Girard

November, 19-20th, 2013

Tuesday, 9:00–9:30, Welcome coffee.

Tuesday, 9:30–12:00, Session I, chair Anne-Catherine Favre.

Some (nonparametric) tests for copulas: Tests of independence, exchangeability, extreme-value dependence, goodness-of-fit, for change-point detection, ...

Ivan Kojadinovic (Université de Pau, France)

Copulas are being increasingly used to model multivariate distributions with continuous margins in fields such as hydrology, biostatistics, actuarial sciences and finance. One of the main issues in practice is which parametric copula family to choose as a model for the unknown true copula. In this talk, several nonparametric tests that can help to reduce the number of candidate parametric copula families will be presented. The main theoretical results underlying the tests will be reviewed and the testing procedures will be described and illustrated. The talk is based on joint work with Axel Bücher, Christian Genest, Mark Holmes, Johanna Neslehová, Johan Segers and Jun Yan.

Pair-copula constructions - even more flexible than copulas

Kjersti Aas (Norwegian Computing Center, Norway)

A copula is a multivariate distribution with standard uniform marginal distributions. While the literature on copulas is substantial, most of the research is still limited to the bivariate case. However, some years ago hierarchical copula-based structures were proposed as an alternative to the standard copula methodology. One of the most promising of these structures is the pair-copula construction.

Pair-copula constructions are also called regular vines. The modeling scheme is based on a decomposition of a multivariate density into a cascade of pair copulae, applied on original variables and on their conditional and unconditional distribution functions. Each pair copula can be chosen arbitrarily and the full model exhibit complex dependence patterns such as asymmetry and tail dependence. In this talk I will first give an introduction to pair-copula constructions and then significant recent advances in this field will be reviewed.

Patchwork copulas

Fabrizio Durante (Free University of Bozen-Bolzano, Italy)

We present a general view of patchwork constructions of copulas that encompasses previous approaches based on similar ideas (ordinal sums, gluing methods, piecing-together, etc.). Practical applications of the new methodology are connected with the determination of copulas having specified behaviour in the tails, such as upper comonotonic copulas. Some consequences in the approximation of copulas will be also discussed.

Tuesday, 12:00–14:00, Lunch.

Tuesday, 14:00–15:40, Session II, chair Juliette Blanchet.

Strong mixing properties of max-stable random fields

Clément Dombry (Université de Franche-Comté, France)

In this talk, we review the ergodic and mixing properties of stationary max-stable random processes established by Stoev and Kabluchko & Schlather and we present new results about strong mixing properties (in the sense of beta-mixing). As an application, we show that these new estimates entail a central limit theorem for stationary max-stable random fields and we derive the asymptotic normality of three simple estimators of the pair extremal coefficient.

Statistical Modelling of Spatial Extremes

Mathieu Ribatet (Universités Lyon 1 & Montpellier 2, France)

The areal modelling of the extremes of a natural process such as rainfall or temperature is important in environmental statistics; for example, understanding extreme areal rainfall is crucial in flood protection. This article reviews recent progress in the statistical modelling of spatial extremes, starting with sketches of the necessary elements of extreme value statistics and geostatistics. The main types of statistical models thus far proposed, based

on latent variables, on copulas and on spatial max-stable processes, are described and then are compared by application to a dataset on rainfall in Switzerland. Whereas latent variable modelling allows a better fit to marginal distributions, it fits the joint distributions of extremes poorly, so appropriately-chosen copula or max-stable models seem essential for successful spatial modelling of extremes.

Tuesday, 15:40–16:00, Coffee break.

Tuesday, 16:00–17:40, Session III, chair Marianne Clausel.

Bayesian modelling of financial extremes in high dimension

Thomas Opitz (Université Montpellier 2, France)

Extreme value analysis is based on max-stable limits for component-wise maxima and the corresponding peaks-over-threshold-stable limits for exceedances above a high threshold. First, a review of theoretical results, inference and simulation algorithms for the limit distributions of asymptotically dependent multivariate elliptical distributions is given. The dependence structure of the resulting extremal elliptical model is characterized by a correlation matrix and a concentration parameter, thus allowing for a flexible parametrization. In particular, such limits arise for elliptical t distributions with the concentration parameter corresponding to the degree of freedom; hence this model is often also labeled the extremal-t model in the literature. An efficient likelihood approach, based on threshold exceedances and a partial censoring scheme for vector components falling below their corresponding threshold, is presented. In practice, a comparison between the extremal model, estimated from threshold exceedances, and a global t-copula, estimated from the whole sample with robust methods, can reveal if dependence among extreme events differs substantially from global behavior. A Bayesian approach is developed which does not impose any parametric restriction on the correlation matrix. We propose modeling of the extremal dependence in high-dimensional financial loss data, here exemplified for European stocks from the finance sector.

Dirichlet mixtures for multivariate extremes: model re-parametrization and Bayesian inference with censored data

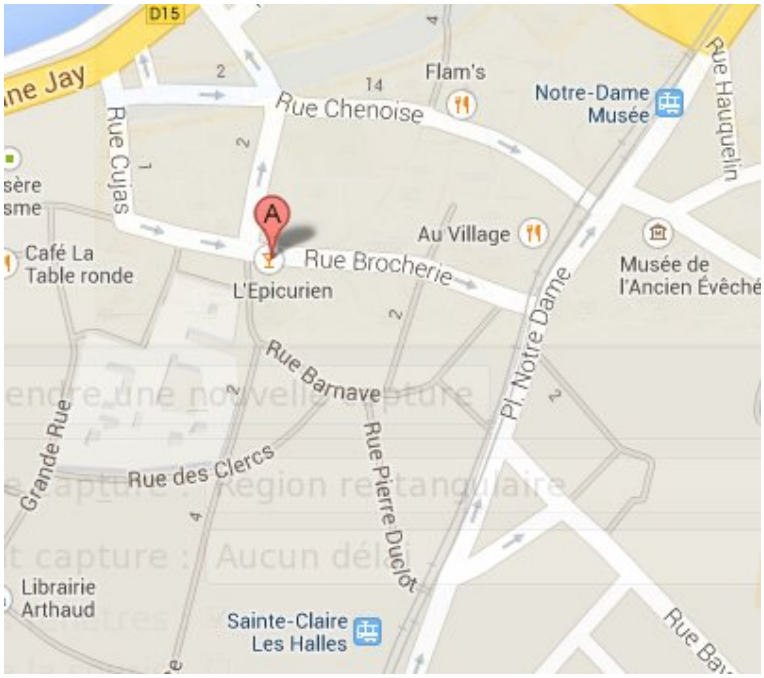
Anne Sabourin (Telecom ParisTech, France)

The dependence structure of multivariate extremes, defined as random vectors which jointly exceed large thresholds, can be characterized, up to marginal standardization, by an angular measure on the simplex, only subject to first moment constraints. Estimating

the angular measure is thus, by nature, a non parametric problem. Finite Dirichlet mixtures can be used to approach weakly such angular measures but, in practice, the moment constraints make Bayesian inference very challenging in dimension greater than three. We present a re-parametrization of the Dirichlet mixture model, in which the moment constraints are automatically satisfied. This allow for a natural prior specification as well as a simple implementation of a reversible-jump MCMC. Posterior consistency and ergodicity are veried. We illustrate the methods with a four-variate streamflow dataset, including historical information (the earliest flood has been recorded in 1604), which results in censored and missing data. Advantage is taken of the conditioning and marginalization properties of Dirichlet distributions to resolve censored likelihood issues within a data augmentation framework. This is joint work with Philippe Naveau, Benjamin Renard and Anne-Laure Fougères.

Tuesday, 20:00, Dinner at restaurant *L'épicurien*

- All invited speakers and registered people are invited.
- The restaurant *L'épicurien* is located at 1, place aux Herbes, Grenoble. You may have a look at the website `http://www.lepicurien-grenoble.com`.
- How to come: Take the tramway (line B) and stop at *Notre-Dame Musée*.



Wednesday, 9:00–10:40, Session IV, chair Clémentine Prieur.

Multivariate return periods in earth sciences: a copula approach

Gianfausto Salvadori (Università del Salento, Italy)

The concept of Return Period is adopted in Earth Sciences (e.g., hydrology, coastal and off-shore engineering, volcanology, seismology, ...) as a common criterion for design purposes and sizing works, and may also provide a means for risk analysis and assessment. According to several Authors, the Return Period is analogous to the Value-at-Risk used in Finance.

The construction of a consistent notion of multivariate Return Period is rather tricky, since many different definitions are possible. Similarly, the identification problem of design events in a multivariate context is of fundamental importance but, at the same time, is of troublesome nature.

This talk will show a possible construction of a coherent notion of multivariate Return Period, exploiting Copulas and the Kendall's measure. In addition, suitable strategies for the identification of multivariate critical occurrences will be presented. Finally, the use of the multivariate Return Period for the real-time assessment of multidimensional problems will be illustrated.

A nonparametric class of non-exchangeable copulas in high dimension and its estimation

Gildas Mazo (Inria Grenoble Rhône-Alpes, France)

The concept of copulas is a useful tool to model multivariate distributions. While many families of copulas have been proposed in the bivariate case, much fewer has been done in higher dimension. Due to their tractability, Archimedean copulas are widely used in practice. They assume that the dependence structure at play is exchangeable. However, because they have only a few parameters, they are not flexible. In this communication, we propose a class of nonparametric copulas, including Extreme-value copulas, which exhibit a good balance between flexibility, parsimony and tractability. The inference, simple and fast, is done through an M-estimation procedure and the asymptotic properties are derived. An application with hundreds of variables illustrates the applicability of the proposed class.

Wednesday, 10:40–11:00, Coffee break.

Wednesday, 11:00–12:40, Session V, chair Stéphane Girard.

Nested Archimedean copulas: structure estimation and goodness of fit

Nathan Uyttendaele (Université Catholique de Louvain, Belgique)

Nested Archimedean copulas (NACs) are multivariate distributions defined by a tree structure and a collection of univariate functions called generators, one for each internal node of the tree structure. Inspired by previous works on the matter, various new approaches to estimate the structure of a NAC are presented. The generators across the estimated structure are marginally estimated. As there exist constraints on the generators that marginal estimation cannot capture, there is no guarantee the resulting NAC will be a true multivariate distribution. Yet this estimated NAC can be conveniently used to assess how well the model fits the data and several examples are given.

Exchangeable exogenous shock models and a copula-characterization of self-decomposability on the half-line

Steffen Schenk (Technische Universität München, Germany)

We consider multivariate distribution functions whose arguments are ordered, idiosyncratically distorted, and then multiplied. Necessary and sufficient conditions on the distortions to yield a proper multivariate distribution function are given, and the distribution functions are shown to characterize a large family of exchangeable exogenous shock models. The vector of exceedance times of an additive subordinator across independent exponential trigger variables is shown to constitute an important special case of the considered family, and yields a second probabilistic model for a large subclass. In particular, when the subordinator is not only additive but also self-similar, a novel analytical characterization for the Laplace transform of a self-decomposable law on the half-line via copulas can be deduced. This is joint work with Jan-Frederik Mai and Matthias Scherer.