Precipitation areal-reduction factor estimation using an annual-maxima centered approach

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Abstract

The adjustment of precipitation depth of a point storm to an effective (mean) depth over a watershed is important for characterizing rainfall-runoff relations and for cost-effective designs of hydraulic structures when design storms are considered. A design storm is the precipitation point depth having a specified duration and frequency (recurrence interval). Effective depths are often computed by multiplying point depths by areal-reduction factors (ARF). ARF range from 0 to 1, vary according to storm characteristics, such as recurrence interval; and are a function of watershed characteristics, such as watershed size, shape, and geographic location. This paper presents a new approach for estimating ARF and includes applications for the 1-day design storm in Austin, Dallas, and Houston, Texas. The approach, termed “annual-maxima centered,” specifically considers the distribution of concurrent precipitation surrounding an annual-precipitation maxima, which is a feature not seen in other approaches. The approach does not require the prior spatial averaging of precipitation, explicit determination of spatial correlation coefficients, nor explicit definition of a representative area of a particular storm in the analysis. The annual-maxima centered approach was designed to exploit the wide availability of dense precipitation gauge data in many regions of the world. The approach produces ARF that decrease more rapidly than those from TP-29. Furthermore, the ARF from the approach decay rapidly with increasing recurrence interval of the annual-precipitation maxima. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The adjustment of precipitation depth of a point storm to an effective (mean) depth over a watershed is important for characterizing rainfall-runoff relations and for reducing precipitation volume for hydraulic design when design storms are considered.

A design storm is the point precipitation having a specified duration and frequency or recurrence interval (e.g. the 24-h 100-year design storm). Effective depths often are computed by multiplying point depths by “depth-area correction factors” or “areal-reduction factors” (ARF). ARF are generally defined as the ratio of the mean precipitation depth over a watershed resulting from a storm to the maximum point depth of the storm. ARF range from 0 to 1; vary according to storm characteristics, such as recurrence interval; and are functions of watershed characteristics, such as watershed size, watershed shape, and geographic location.
The distribution of precipitation shows large variation in both space and time. Therefore, depth-area studies have required extensive statistical inference because the underlying databases typically are not large. For this paper, however, three large precipitation databases are available. Hence, a new approach was designed to exploit these data. Large amounts of precipitation data facilitate an approach that specifically considers the distribution of precipitation concurrent with and surrounding an annual precipitation maxima. Because the approach is based solely on annual-precipitation maxima and concurrent precipitation, it is termed an “annual-maxima centered” approach. The approach relies on less statistical inference than previous studies and might be more applicable in design practice. Unlike previous studies, the approach is straightforward, computationally simple, and does not require the prior spatial averaging of precipitation, explicit determination of spatial correlation coefficients, nor explicit definition of a representative area of a particular storm in the analysis. The annual-maxima centered approach is presented by example for 1-day design storms with applications for Austin, Dallas, and Houston, Texas.

1.1. Previous studies

The estimation of effective precipitation depth for design storms is a complex problem that has been approached in a variety of ways (see US Weather Bureau, 1957; Miller et al., 1973; Rodríguez-Iturbe and Mejía, 1974a,b; Myers and Zehr, 1980; Omolayo, 1993; Bacchi and Ranzi, 1996; Sivapalan and Blöschl, 1998; and references there in). This section briefly outlines some of the various methods used for ARF estimation and for calculation of effective precipitation depth ($Z_E$). Myers and Zehr (1980) point out that ARF is the ratio of two different expectations (areal average and point precipitation depths) and generally is not intended to describe the spatial or temporal variability of design storms, or to describe the complex morphology of individual storms, or to provide the basis for stochastic simulation.

Two types of ARF are commonly used. The first is known as the “geographically fixed” or “fixed-area” ARF that relates the point depth to the average depth of concurrent precipitation over a specified fixed area. The geographically fixed ARF is based on magnitude and frequency analysis of a time series of annual maximum mean precipitation computed for a given and fixed area. An assumption of probability equivalence generally is made between the frequency of the point precipitation and the frequency of the areal precipitation (e.g. the $T$-year point precipitation generates the $T$-year volume of precipitation, where $T$ is the return period). The second type of ARF is known as “storm-centered,” and is often associated with the calculation of $Z_E$ for individual storms. The average storm depth is commonly derived from integration of contour lines of depth divided by the maximum depth recorded in the storm. Sivapalan and Blöschl (1998) report that a storm-centered ARF is “usually somewhat smaller than [geographically] fixed [ARF]”. The discussion in this section is restricted to geographically fixed ARF because storm-centered approaches have not seen widespread application for several reasons including difficult implementation on multi-centered storms.

Perhaps the most common sources of ARF for the United States are Technical Paper 29 (TP-29) by the US Weather Bureau (1957), NOAA Atlas 2 by Miller et al. (1973), and Zehr and Myers (1984). TP-29 provides ARF for areas ranging from 0 to about 1000 km$^2$ for storm durations less than or equal to 24 h. The TP-29 analysis is based on precipitation-monitoring networks east of the Mississippi River, but TP-29 is commonly used outside this region. TP-29 defines an ARF as the ratio of the mean annual maxima of areal precipitation to the mean annual maxima of point precipitation. The area covered by $n$ stations is assumed equal to the total area of $n$ circles, each having a radius equal to one-half the average station separation distance. Area has a large influence on TP-29 correction factors; however, frequency considerations could not be reliably specified from the short periods of record for the available precipitation stations. Accordingly, TP-29 presents a single ARF-area curve based on 2-year recurrence intervals, which was judged applicable for all return periods up to 100 years. Though widely recognized as a tenuous assumption, TP-29 assumes that the relation between depth and area is not influenced by the frequency (recurrence interval) of the point precipitation. TP-29 implicitly equates the frequency of the point precipitation to the frequency of the areal precipitation.
Rodriquez-Iturbe and Mejia (1974a,b) developed a general method for converting any point precipitation to an effective precipitation. Their method is primarily oriented towards solutions for problems of distributing precipitation from multiple inputs in a rainfall-runoff model setting, estimating long-term mean effective precipitation, and estimating effective depths for individual storms. The ARF of Rodriquez-Iturbe and Mejia depends solely on the expected correlation coefficient for a characteristic correlation distance. The characteristic correlation distance is the mean separation distance between two randomly chosen points in the area. The Rodriquez-Iturbe and Mejia correction factor is:

$$\text{ARF} = \sqrt{E[\rho(d)]},$$  \hspace{1cm} (1)

where, \(E[\rho(d)]\) is the expected correlation coefficient for the characteristic correlation distance.

Although the ARF of Eq. (1) is simple, estimation of \(E[\rho(d)]\) is a difficult problem fraught with many uncertainties (Rodriquez-Iturbe and Mejia, 1974a,b). Like many of the sources listed above, the Rodriquez-Iturbe and Mejia approach equates the frequency of the point precipitation to the effective precipitation frequency. Although it provides an extensive framework for transforming point depths to effective precipitation, the Rodriquez-Iturbe and Mejia approach is not specifically oriented toward estimation of the areal distribution of design (extreme) storms. To our knowledge, the Rodriquez-Iturbe and Mejia (1974a,b) method has not been extensively used for estimating the \(Z_E\) of design storms.

Another complex and computationally intensive method for ARF calculation is presented in Myers and Zehr (1980). Their model is based on extensive statistical inference to compensate for a lack of precipitation data. Another variant of ARF calculation is proposed by Sivapalan and Bloeschl (1998). They point out that the variance of a point precipitation process is greater than the variance of an areal process. The ratio of the areal variance to point variance is termed a “variance-reduction factor”, and their ARF are based on this ratio. The variance-reduction factor is estimated from the spatial correlation of precipitation. The spatial correlation is defined by all the precipitation data available within a network for the duration of interest. Their approach finds that ARF depend heavily on the recurrence interval of the point precipitation.

Though the methods of investigation vary, the ARF from the various sources all show similar behavior. ARF are at or near unity for very small areas. For increasing areas, ARF for a given duration and recurrence interval decrease (decay) in an exponential-like fashion. ARF decrease more rapidly for short duration storms (such as 30 min) than for long durations (such as 1 day). Furthermore, as the recurrence interval (intensity) of the point precipitation increases, ARF decrease more rapidly.

Some general comments are needed about the previous approaches. In TP-29, determination of a representative area of a precipitation event or network is arbitrary, and watershed shape is not considered. For example, a long narrow watershed of given area has the same TP-29 ARF as a circular watershed of equal area. The Rodriquez-Iturbe and Mejia ARF are very simple to compute, but computation of the expected separation distance between two random points in a watershed often requires numerical analysis; and the correlation coefficient is derived from analysis of all precipitation for the duration of interest regardless of the various mechanisms by which the storms were generated. The Myers and Zehr (1980) approach to ARF calculation is computationally complex and difficult to implement in design practice. The Sivapalan and Bloeschl ARF has perhaps the most applicability of existing approaches to extreme precipitation events; however, in their method a probability equivalence between point and areal precipitation is a central assumption as is the characteristic correlation length. The correlation length is derived for particular storm events or types (thunderstorms have smaller correlation lengths, whereas, tropical storms would have larger correlation lengths). Determination of correlation lengths for design applications is problematic. The annual-maxima approach aims to mitigate the application difficulties and shortcomings of other approaches by using a straightforward approach for use in densely gauged regions.

2. Methods

2.1. Annual maxima-centered ARFs

The annual-maxima centered approach requires the completion of several steps. Each step is discussed in
sequence after the theory is introduced. The steps of
the approach are:

1. **Sample Ratios**—Compute for every annual
maxima in a database, the ratio of the annual
maxima depth to concurrent precipitation (same
day precipitation for this paper) and the separation
distance between the two precipitation gauges.

2. **Empirical Ratio Relations**—Derive from the
sample ratios for a selection (design) criteria,
such as those ratios surrounding a 5-year or greater
annual maxima, a preliminary description of the
relation between criteria-conditioned sample-ratio
values and separation distance (ratio relation).

3. **Estimated Ratio Relations**—Define the ratio rela-
tion through some function or set of functions fitted
to the empirical ratio relation. This step provides an
average line (linear or curvilinear) that is the
expected ratio value for a given distance from the
annual maxima. The line must be fit to pass through
unity at zero distance.

4. **Areal-Reduction Factors**—Compute ARF for a
user defined area and design criteria by spatially
integrating the estimated ratio relation for the
design criteria from step 3.

The last step is an elementary calculation. Though this
paper is ultimately about ARF, the most important
element of the annual-maxima centered approach is
the estimated ratio relation of step 3. The area over
which the design storm occurs is user specified and is
needed only at the very end of analysis. This is an
important feature of the approach because definition
of the area represented by individual storms or by a
precipitation-monitoring network for analysis leading
to ARF is often difficult.

2.2. Theory

The precipitation volume ($V$) over a watershed ($W$)
surrounding a point storm (such as a $T$-year, 24-h
storm) can be expressed as a spatial integral:

$$V = \int_W dV = \int_W [Z(T, x, y)] \, dx \, dy,$$  \hspace{1cm} (2)

where, $Z(T, x, y) = \text{the precipitation field associated}
with a point depth. $Z(T, x, y)$ can be formulated as the
product of $Z_T$ (the design storm precipitation) and
some unknown spatial relation function, $S_T^{\text{rel}}(x, y)$,
which describes the spatial structure of the storm.

$S_T^{\text{rel}}(x, y)$ is dimensionless, continuous, non-negative,
and unbounded above. Additionally, it is necessary
and sufficient for only the first moment (the mean)
to exist for every $(x, y)$ location. Eq. (2) is rewritten as:

$$V = \int_W Z_T S_T^{\text{rel}}(x, y) \, dx \, dy. \hspace{1cm} (3)$$

Either Eqs. (2) or (3) is sufficient to characterize the
storm volume because the precipitation varies for
every location of the watershed. It is assumed that
the largest potential volume of a storm occurs when
the storm is centered over the centroid of the
watershed. Therefore, by conservative definition, $Z_T$,
the depth of the design storm, is located at the centroid
$(x_c, y_c)$ of the watershed. Furthermore, if it is assumed
that storm orientation over the watershed is not impor-
tant-storms are assumed to be generated by an isotro-
pic spatial process—$S_T^{\text{rel}}$ becomes symmetric and can
be generalized by the separation distance ($r$) between
the centroid and the location of $(dx \, dy)$. The separation
distance is defined as:

$$r = \sqrt{(x - x_c)^2 + (y - y_c)^2}. \hspace{1cm} (4)$$

Accordingly, Eq. (3) can be rewritten as:

$$V = \int_W Z_T S_T^{\text{rel}}(r) \, dx \, dy. \hspace{1cm} (5)$$

$S_T^{\text{rel}}$ in this form describes the spatial structure of a
storm radiating away from $Z_T$ at the centroid of the
watershed, which implies that $S_T^{\text{rel}}$ must be unity at zero
distance.

Though the storm volume model (Eq. (5)) is suf-
ficient for volume characterization, it is lacking some
real-world conformance. Real-world conformance is
extended by recognizing that no two centroid-
centered storms capable of producing a $Z_T$ would be
identical. Thus, $S_T^{\text{rel}}(r)$ is a random variable for each
and is unique for each storm. More specifically, the
spatial distribution of concurrent rainfall with $Z_T$ can
be considered a random variable for each $r$. Storm
volume is rewritten as:

$$V_i = \int_W Z_T S_T^{\text{rel}}(r, F) \, dx \, dy, \hspace{1cm} (6)$$

where, $V_i$ is the realization of storm volume, $F$ the
cumulative probability, and $S_T^i$ a random variable for a cumulative probability $F$ at each $r$.

The subscript “i” has been added to $V$ in order to indicate that with each stochastic integration of Eq. (6), a single realization of the storm volume is generated. With this storm volume characterization, it is possible to generate many realizations of $V_i$, which form a series of random variables $(V_1, V_2, \ldots, V_n)$ of the storm volume. The series of $V_i$ will allow investigation of the distribution of $V$ and formulation of its inverse cumulative distribution function $V(F)$, $0 \leq F \leq 1$.

However, for typical use in applications involving ARF, the actual distribution of $V(F)$ is not required. Only the expected value of $V(F)$ is needed. The expected value of $V(F)$ is the expected volume of design storm $V_T$ for watershed $W$ and the $T$-year recurrence interval. The expected value of the design storm volume ($V_T$) is computed as:

$$V_T = E[V(F)] = E\left[ \int_W Z_T S_T^i(r, F) \, dx \, dy \right]$$

$$= \int_W Z_T E[S_T^i(r, F)] \, dx \, dy,$$

(7)

where the expectation operator, $E[ ]$ is

$$E[S_T^i(r, F)] = \int_0^1 S_T^i(r, F) \, dF = S_T(r).$$

(8)

From Eq. (8), a single expected value of the spatial relation function, $S_T(r)$, exists; and this value is a function of the recurrence interval and the distance from the centroid. $S_T(r)$ describes the average spatial structure of the precipitation concurrent with, or radiating from, the point of an annual-precipitation maxima. $S_T(r)$, in other words, is the expected value of the ratio between the depth at some location a distance $r$ from the point of the design storm and the depth of the annual maxima. $S_T(r)$ is referred to here as the “ratio relation”.

$S_T(r)$, as defined by Eq. (8), requires the assumption that the covariance or cross correlation of concurrent precipitation at two non-centroid locations is insignificant. The feasibility of ignoring this spatial correlation is an important assumption of the approach. Because our objective is to describe the expected volume over a watershed and not the stochastic simulation of storms, concern over ignoring the spatial covariance structure is mitigated. Regarding Eqs. (7) and (8), only one design volume $V_T$ exists for a given watershed and design criteria. The expected volume for a design storm is:

$$V_T = Z_T \int_W S_T(r) \, dx \, dy.$$  

(9)

A final observation is that $V_T$ is the expected volume for design storm $Z_T$ and is not restricted to being the $T$-year design volume. A probability equivalence between volume and design storm depth is not required, though an equivalence must occur in the limit as watershed area becomes zero.

The ARF$_T$ for $Z_T$ is expressed as:

$$Z_{Ei} = \frac{V_i}{\text{AREA}},$$

(10)

$$\text{ARF}_{Ti} = \frac{Z_{Ei}}{Z_T}.$$  

(11)

Realizations of $Z_{Ei}$ or ARF$_{Ti}$ analogous to $V_i$ could be generated. Because of linearity, either $V_T$, $Z_{Ei}$, or ARF$_{Ti}$, is needed to compute the others. The expected (design) values of $Z_{Ei}$ (effective depth for a recurrence interval) and ARF$_T$ are therefore computed as:

$$Z_{Ei} = \frac{V_T}{\text{AREA}} = \frac{Z_T \int_W S_T(r) \, dx \, dy}{\int_W dx \, dy},$$

(12)

$$\text{ARF}_T = \frac{Z_{Ei}}{Z_T} = \frac{1}{Z_T} \frac{Z_T \int_W S_T(r) \, dx \, dy}{\int_W dx \, dy} = \frac{\int_W S_T(r) \, dx \, dy}{\int_W dx \, dy}.$$  

(13)

Often it is easier to consider ARF-area relations directly, therefore, for a circular area Eq. (13) can
be rewritten as:

\[
\text{ARF}_T = \frac{Z_{ET}}{Z_{T}} = \frac{1}{Z_{T}} \int_{R}^{r} 2\pi r S_T(r) \, dr \int_{R}^{r} 2\pi r \, dr = \int_{0}^{R} \frac{2\pi r S_T(r) \, dr}{R^2},
\]

where, \(2\pi r\) is the circumference of a concentric circle at radius (separation distance) \(r\) within the circular watershed, and \(R\) the maximum radius of the circular watershed or the limit of the integration.

We chose circular areas in order to visualize ARF-area relations. The integration is easiest and most general. For brevity, \(r\) is dropped hereafter from, though explicitly understood in, reference to \(S_T(r)\).

### 2.3. Daily precipitation data sources

The annual-maxima centered approach was designed for use in regions of the world having dense precipitation data. Three such regions include the area around the cities of Austin, Dallas, and Houston, Texas. This paper presents application of the annual-maxima centered approach for these cities. The “city databases” of daily precipitation were aggregated from several precipitation-monitoring networks in the localities. For a 15,600-km² area around Austin, two daily precipitation networks were identified: 25 National Weather Service (NWS) stations and 83 City of Austin (AUS) stations. For a 21,000-km² area around Dallas, two daily precipitation networks were identified: 58 NWS stations and 45 City of Dallas (DAL) stations. For a 35,800-km² area around Houston, three daily precipitation networks were identified: 64 NWS stations, 84 Harris County Office of Emergency Management (HAR) stations, and 45 US Geological Survey Houston Urban Program (HURP) stations.

The stations in each network have varying periods of record and record completeness. In general, the NWS stations have the longest periods of record and the most complete record. Some NWS stations have record lengths in excess of 80 years. In general, the AUS stations have start dates from 1988 to 1990 and were operated through 1996. The DAL stations generally operated from 1991 through 1997. The HAR stations have various start dates in the late 1980s and operated through 1997. The HURP stations have various periods of record, but in general, the records start between 1965 and 1975 and end between 1984 and 1989.

The NWS stations recorded a “trace” as part of the non-zero record. These data were assumed to be zero, subsequently reset, and included in the analysis. Trace data amounts are not reported by the other networks. Individually, the Austin, Dallas, and Houston databases have about 248,000; 429,000; and 688,000 values of daily precipitation.

Each network in the three databases was extensively evaluated to test for its performance and its compatibility with other networks. A summary of the evaluation results is needed in the context of describing the annual-maxima centered approach and is deferred until Section 3.1.

### 2.4. Sample ratios

The ratio relation Eq. (8) is the ratio of the precipitation \(Z(r)\) at a distance \(r\) away from a point of the annual maxima to the point precipitation \(Z_T\):

\[
\frac{Z(r)}{Z_T} = \frac{1}{Z_T} \int_{r}^{R} 2\pi r S_T(r) \, dr \int_{r}^{R} 2\pi r \, dr = \int_{0}^{R} \frac{2\pi r S_T(r) \, dr}{R^2},
\]

when a sufficiently large number of ratios \(Z(r)/Z_T\) from a densely spaced precipitation-monitoring network are available, these “sample ratios” provide the basis for estimating \(S_T\). The sample ratios are easily computed by determining the annual maxima for each year and each station and dividing this value by each precipitation depth recorded on the same day for each surrounding station. Additional information needed for each ratio were identifying which network was used in the numerator, which network was used in the denominator, and the recurrence interval of the annual maxima (denominator). A sample ratio database was created for each city.

Two prominent assumptions are necessary for the sample ratios to be useful. The first is that \(S_T\) is assumed to be stationary over the study area. This requires that the moments (e.g. mean, standard deviation) of \(S_T\) for a specified \(r\) be position invariant. The second assumption is that the actual temporal distribution of precipitation within a day is unimportant. For example, the spatial distribution of a 2-h or a 20-h storm, which produces a daily annual maxima, are assumed comparable, although it is recognized
that the meteorologic mechanisms generating each could differ.

The degree of missing daily precipitation values has important ramifications on assuming whether or not the observed annual maxima for a given year is approximately equal to the true annual maxima. The observed annual maxima for a given year is necessarily too small if the true annual maxima occurred on a day with missing record (consideration of the timing of the annual maxima during a 24-h period is ignored). If the number of missing days is too large, then the apparent annual maxima from an incomplete year is too small. Testing (results not presented here) indicated that allowing 10 missing days per year provided an appropriate trade off between observing the true annual maxima and not having an annual maxima (even if just apparent) to perform subsequent analysis.

A representative sample of the Dallas sample ratios is shown in Fig. 1. The sample ratios are for any annual precipitation maxima—that is annual maxima for any recurrence interval \(1 \leq T \leq \infty\). The sample ratios plotted represent a small random subset (only about 3200 of 42,000 ratios with separation distances less than 80 km). It is evident from Fig. 1 that the variability of the ratios is large. Numerous ratios are zero, which are increasingly more likely to occur as \(r\) increases. Ratios larger than one are not uncommon—a fact that matches the physical reality that locations other than the point coincident with the annual maxima for a particular precipitation gauge can have larger concurrent depths.
The sample ratio databases were statistically summarized for each mile-wide window of separation distance. The mean ratios for each mile-wide window provide the ratio relation. To provide an unbiased separation distance for graphical representation and further analysis, the mean distance for each mile-wide interval was computed. Computing the mean ratio for each mile-wide window is equivalent to computing the mean sample ratio within each mile-wide concentric ring surrounding an annual maxima. We chose a 1.61-km (1-mile) wide window after testing values between 0.80 and 8 km and chose to limit distances to between 0 and 80 km. The empirical $S_T$ is defined by the mean ratio for each mile-wide window of separation distance after the sample ratios are conditioned according to recurrence interval. The term conditioned means selecting and using those sample ratios concurrent with a $T$-year or greater annual maxima. Because very few annual maxima equal to or nearly equal to the $T$-year event are available, it is necessary that the conditioning be a cumulative type. For example, those ratios generated by a 2-year or greater annual maxima define the empirical $S_2$.

3. Ratio relations

Ratio relations ($S_T$) eventually provide the basis for the calculation of annual-maxima centered ARF ($S_T$ (Eq. (13))). Though separation distances larger than 80 km were available from the databases, the maximum $r$ presented in this paper was limited to 80 km. An $r$ of 80 km corresponds to a circular area of about 20,100 km$^2$, which is far in excess of the drainage areas for which ARF values are typically needed or for which an ARF would be expected to be appropriate. The following section briefly describes some evaluations of individual network performance and network compatibility.

3.1. Database evaluation

Before steps 2–4 of the approach can be completed, an assessment of network compatibility was necessary. Each network is operated by a different agency. Considerable differences in operation methods exist and include, but are not limited to, differing instrument styles (recording or non-recording), instrument types (tipping bucket or weighing), instruments (manufacturers and models), instrument heights (above ground), instrument exposure (airports versus residential backyards), reporting times (midnight to midnight or 8:00 a.m. to 8:00 a.m.), and instrument calibration (maintenance and accuracy of leveling). Groisman and Legates (1994) provide a detailed assessment of the accuracy of precipitation data and discuss many of the factors identified above.

An evaluation of how well the stations of each network recorded the annual maxima was based on the comparison of observed annual cumulative probabilities to assumed true values (or quantiles) derived from Asquith (1998). The results of the evaluation are summarized here, further details are provided by Asquith (1999). The evaluation concluded that each network appears to systematically underestimate the assumed true precipitation quantiles. The AUS, DAL, and HAR networks show larger defined–observed probability differences than either the NWS or HURP networks. The AUS network has the largest differences. Factors contributing to the differences might include: effects of missing record, short record length, climatic cycles, instrumentation.

Some underestimation is expected because daily observations of precipitation (fixed-interval recording) will underestimate the true 24-h precipitation depth by about 14% (Weiss, 1964). Asquith (1998) used a bias correction by Weiss (1964). The correction for the bias is only possible on the mean of an annual maxima time series. It is not possible to correct individual annual maxima for the bias; therefore, such a correction was not available for this paper.

A second evaluation was conducted to assess network compatibility of the concurrent daily precipitation and the annual-precipitation maxima. The results of the evaluation are summarized here, but further details are provided by Asquith (1999). The evaluation was based on the empirical $S_2$ derived specifically from each network.

Two types of $S_2$ can be derived from the networks for each city. The first type, intra-network $S_2$ represent mean-sample ratios computed independently from each network without regard to the presence of, or “cross” comparison to, the other networks.
The second type, *inter*-network $S_2$ represent mean-sample ratios computed solely from cross comparison of annual maxima and concurrent precipitation in differing networks.

Separate comparisons of inter-network and intra-network $S_2$ for each city database yielded the same conclusion. Each network appears to have some systematic negative bias, which, if constant for a network, is fortuitously divided out in intra-network $S_2$ but not for inter-network $S_2$. Thus, to mitigate the network biases, only intra-network $S_2$ for each city were used.

The implications of the point and areal evaluations are that differing precipitation-monitoring networks can not be indiscriminately combined to increase the number of sample ratios. It is unknown whether similar results would be seen for other overlapping networks around the world.

### 3.2. Empirical ratio relations

The three large sample ratio databases were used to define various empirical $S_T$. The magnitude, hence frequency, of the storm center (the point of an annual maxima) has considerable influence on the expected (average) decrease of ARF as distance from the storm center increases. This influences the expected areal distribution of precipitation. It is hypothesized that as the maximum point depth (intensity) of a storm increases, the surrounding depths decrease more rapidly. In other words, large or very intense storms are not as widely or as evenly distributed in space as smaller more frequent storms. Each of the three databases support the hypothesis.

Comparisons of empirical $S_T$ for selected recurrence intervals for Austin, Dallas, and Houston are shown in Fig. 2a–c. The recurrence intervals are defined in a cumulative fashion, that is, the empirical $T$-year or greater ratio relation (empirical $S_T$) is derived from only those ratios for which the recurrence interval of the annual maxima of the storm center was equal to or greater than $T$ years and not simply just the $T$-year ratio relation. The empirical $S_T$ for recurrence intervals greater than 5 years are not shown for Austin and Dallas. The record lengths of gauges in the AUS and DAL networks are too short to reliably evaluate the influence of larger recurrence intervals. Though the 5-year recurrence interval is often considered small, its probability level (80th percentile) is considerably far into the upper tail of the distribution. A larger and denser database is available for Houston than for either Austin or Dallas. Hence, the empirical $S_T$ for 10-year or greater recurrence intervals for Houston is also shown. The empirical $S_T$ for recurrence intervals larger than 10 years are not shown.

A trend of more rapid decrease of ratio values with increasing $r$ for larger recurrence intervals is apparent. However, even with the considerable number of stations and a large number of ratios, estimation of expected sample ratio values for $r$ less than about 3 km remains difficult. Recurrence interval might become less important for increasing distances as evidenced by the convergence of $S_T$ for large $r$. All of the empirical $S_T$ appear to level off around ratios of 0.2 to 0.3 for large $r$. The leveling off indirectly represents locations receiving precipitation that is independent of (not affected by distance from) the annual maxima. Finally, the variability of the empirical $S_T$ increases as recurrence interval increases because of sample size reduction because the number of available sample ratios diminishes. A comparison of the empirical $S_T$ for Austin, Dallas, and Houston is presented in Fig. 3, which demonstrates that ARF are not really transferable even within the eastern one-half of Texas let alone across large portions of the United States as is often done with TP-29. The differences between the three $S_2$ are due to several factors that include climate, instrumentation, and periods of data collection.

### 3.3. Estimated ratio relations

The estimated $S_T$ for 1-day design storms near Austin, Dallas, and Houston are presented for only the 2-year or greater recurrence interval. The estimated $S_2$ is more accurate than estimated $S_T$ for greater recurrence intervals because of limited “observation” of rare annual maxima (Section 3.2). The number of sample ratios decreases very rapidly for increasing recurrence interval. However, the other empirical $S_T$ in Fig. 2 might provide reliable assessment of $S_T$ for larger recurrence intervals. The number of sample ratios (and recurrence intervals) for the Austin database decreases from 17,242 (any $T$-year); to 5226 (2-year); and to 1293 (5-year). Likewise, for
Fig. 2. (a–c) Empirical ratio relations for selected recurrence intervals from Austin, Dallas and Houston databases.
the Dallas database, the number of sample ratios decreases from 41,786 (any T-year); to 15,775 (2-year); and to 5146 (5-year). Finally, the number of sample ratios for the Houston database decreases from 69,370 (any T-year); to 21,392 (2-year); to 8536 (5-year); and to 4654 (10-year).

The estimated $S^2$ for Austin, Dallas, and Houston are shown in Fig. 4a–c with the smoothed relation between $S_T$ and $r$. The estimated $S^2$ in the figures are defined by a series of straight-line segments, which were hand-fit to the mean ratios. The data indicate that the “true” underlying relation is curvilinear. Exponential transformations were evaluated but proved to be unsatisfactory, though near exponential decay would be expected. The straight-line segments were used to define $S^2$. For simple comparison, the intra-network medians for each mile-wide interval are shown in Fig. 4a–c. Based on the mean and median, the distribution of ratios is symmetric to left skewed for distances less than about 12 km and becomes increasingly right skewed for larger distances.

The estimated $S^2$ is judged applicable for all recurrence intervals greater than 2 years. However, empirical $S_T$ (Section 3.2) for increasingly large recurrence intervals indicate more rapid decrease of $S^2$ with distance. Consequently, $S^2$ is negatively biased (underestimated) for recurrence intervals near 2 years and positively biased (overestimated) for larger recurrence intervals. Such a situation in general provides conservative (over) estimation of ARF.

4. Areal-reduction factors

The ARF values for circular watersheds were computed through Eq. (14) by using the straight-line segments of $S^2$. The ARF values for large circular areas in the vicinity of Austin, Dallas, and Houston
Fig. 4. (a–c) Estimated 2-year or greater ratio relation from Austin, Dallas and Houston databases. Numbers indicated sample size available for corresponding separation distance interval.
Fig. 5. (a,b) Areal-reduction factors for 2-year or greater 1-day design storms for Austin, Dallas and Houston.
are shown in Fig. 5a. To increase the resolution of the figure, a separate graph showing ARF₂ values for small circular areas is shown in Fig. 5b.

Though we did not attempt to mimic the previously identified approaches, some quick and simple comparisons are possible (Table 1). These comparisons are not intended to represent a rigorous scientific comparison. The ARF for Houston were chosen for the comparison. From the table it is obvious that considerable differences in estimates of ARF exist. Much of the differences are likely due to different geographic locations—hence climate. Neither the ARF from TP-29 (eastern US) or ARF from Sivapalan and Blöschl (1998) (Austria) were derived with precipitation data for Texas. The ARF from TP-29 are considerably larger than those derived here. The TP-29 ARF imply that 24-h storms have a large spatial extent—hence correlation. This is not seen in the Texas precipitation data. The ARF from Sivapalan and Blöschl (1998) are a function recurrence interval and a spatial correlation distance. Their 1.5-year definition is closest to the 2-year or greater limit used here. The table shows the Sivapalan and Blöschl ARF for two arbitrary, but reasonably representative, spatial correlation lengths. A spatial correlation length of about 16 km provides ARF that are close in magnitude to those derived from the annual-maxima centered approach.

<table>
<thead>
<tr>
<th>Area for Houston (km²)</th>
<th>Annual-maxima centered 2-year or greater ARF</th>
<th>TP-29 (US Weather Bureau, 1958)</th>
<th>ARF from Sivapalan and Blöschl (1998)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Spatial correlation length, λ = 16 km, and T = 1.5 years</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Spatial correlation length, λ = 8 km, and T = 1.5 years</td>
</tr>
<tr>
<td>2.59</td>
<td>0.96</td>
<td>1.0</td>
<td>0.96</td>
</tr>
<tr>
<td>25.9</td>
<td>0.88</td>
<td>0.99</td>
<td>0.93</td>
</tr>
<tr>
<td>259.0</td>
<td>0.77</td>
<td>0.94</td>
<td>0.80</td>
</tr>
<tr>
<td>2590.0</td>
<td>0.61</td>
<td>0.91</td>
<td>0.58</td>
</tr>
</tbody>
</table>

5. Conclusions

The annual-maxima centered approach for ARF estimation is a new development in the analysis of the areal distribution of precipitation of design or extreme storms. The approach specifically considers the distribution of concurrent precipitation surrounding annual maxima. The approach was designed to exploit the wide availability of dense precipitation gauge data in many regions of the world. We demonstrate the approach for daily precipitation using precipitation data for the Austin, Dallas, and Houston, Texas areas. Beyond a presentation and application of a new method, the analysis here is believed to be the first depth-area analysis for precipitation-monitoring networks in Texas.

The annual-maxima centered approach is theoretically straightforward, is closely associated with precipitation that occurs concurrent with point design storms, does not require spatial (areal) analysis until the procedure is applied, and finally is easy to apply in design situations. The approach is described and demonstrated in the spirit that it provides an alternative to approaches that are more computationally complex or are based on extensive statistical inference at the expense of requiring considerable databases.

The principal focus of the approach is on the estimation of ARF for design storms, that is, storms having specified recurrence intervals. Recurrence interval has a significant influence on the ratio relations (relation between the ratio of annual maxima to concurrent precipitation depth and separation distance from the annual maxima point) and therefore ARF. As recurrence interval of an annual maxima increases, its associated ARF have increasingly quicker decay characteristics with distance or area. The results represented here were limited to those for the 2-year or greater design storm for brevity.

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