Multivariate Extreme Value Models

A Gentle Introduction

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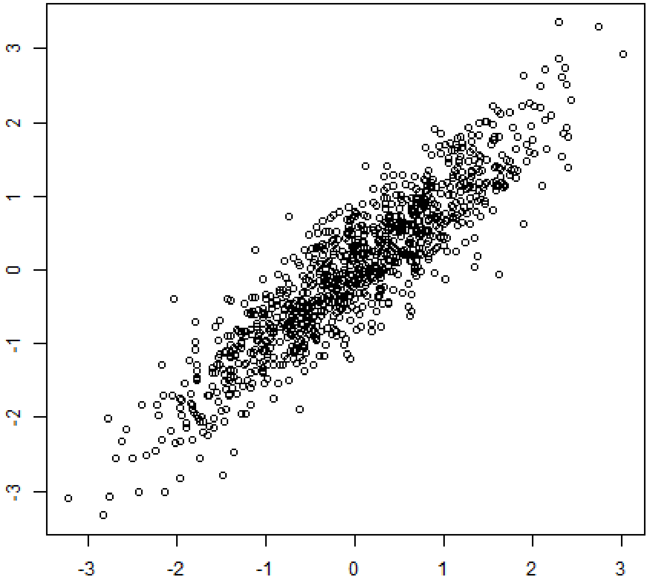
November 20, 2012
Journée de rentrée des doctorants en statistique



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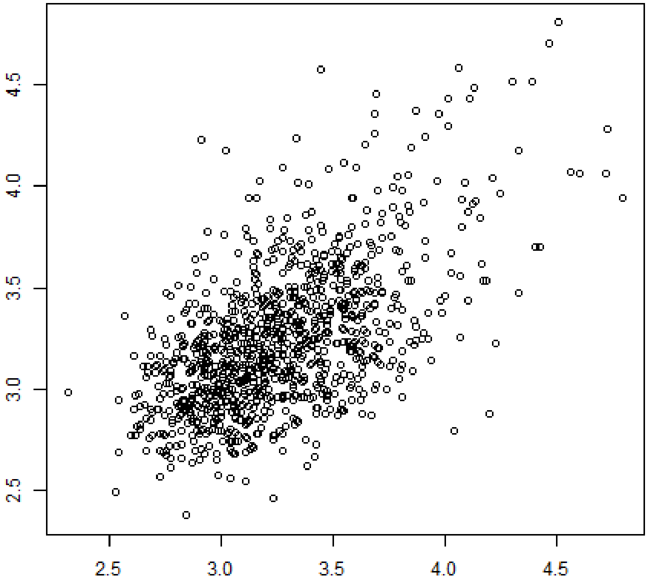
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Mathematical Formulation

Let $X_1, \dots, X_n \overset{iid}{\sim} F$. Then

$$M_n \sim F^n.$$

If

$$M_n \to \text{ some distribution, ie}$$

$$F^n(\cdot, \cdot) \to G(\cdot)$$

we say that G is a **Extreme Value Distribution.**

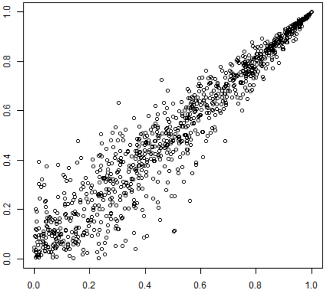
- when $F \sim$ Normal$(\rho < 1)$,

$$G = G_1 G_2$$

Independence!

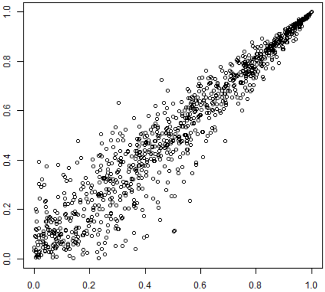
Another Example

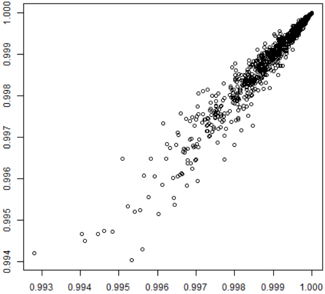
$C^{n}(\cdot, \cdot; \theta)$ $\longrightarrow$ $G(\cdot, \cdot, \theta)$



Another Example

$C^{n}(\cdot,\cdot;\theta)$ $\longrightarrow$ $G(\cdot,\cdot,\theta)$





Theory: Extreme value distribution G

Two equivalent representations

- max-stable:

$$G^n(nx, ny) = G(x, y)$$

- spectral:

$$G(x, y) = \exp - \int_{[0,1]} \max\left(\frac{t}{x}, \frac{1-t}{y}\right) h(t) dt$$

where

$$\int_{[0,1]} t h(t) dt = 1/2$$

NOT equivalent in practice!

Block-Maxima Approach

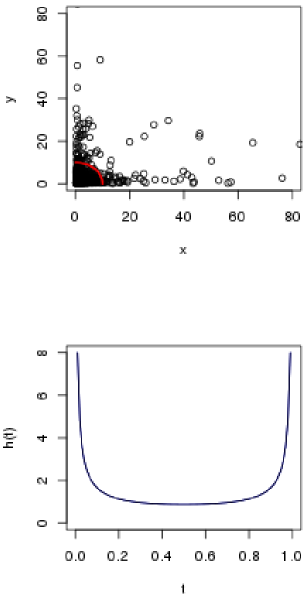
G is the distribution of *maxima*:

$$\left(M_n^{(1)}, M_n^{(2)}\right) \approx G$$

- must have a sample of maxima
  - example: rainfall annual maxima
- can compute probability of rare events like

$$\begin{aligned} P(\text{at least a flood somewhere}) &\approx \\ P(M_n^{(1)} > x \text{ or } M_n^{(2)} > y) &\approx \\ 1 - G(x, y) & \end{aligned}$$

Spectral Approach



Inference

I shall focus on maximum-likelihood inference.

- Block-maxima approach: derive likelihood and maximize it
- Spectral approach: points are thrown according to a Poisson point process:

$$L(\theta; t_i) \propto \prod h(\theta; t_i)$$

- dimension 2 OK!

dimension$>$ 2?

Example

- Most Simple model (Gumbel logistic):

$$-\log G(x_1, \ldots, x_d) = \left(\sum_{i=1}^{d} x_i^{-1/\alpha} \right)^{\alpha}$$

- More (Too) complicated model (Tawn asymmetric logistic):

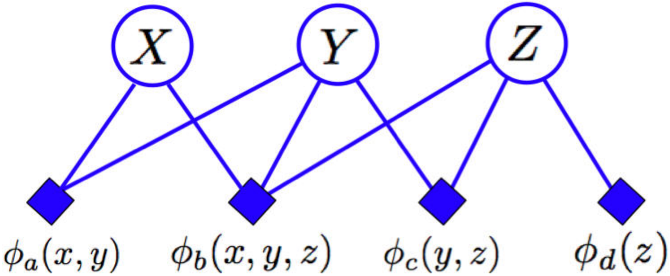
$$-\log G(x_1, \ldots, x_d) = \sum_{B \in \mathcal{B}} \left(\sum_{i \in B} (t_{i,B} x_i^{-1})^{1/r_B} \right)^{r_B}$$

- Compute the likelihood function (and its gradient): *Left as an exercise!*

One Possibility

Cumulative Distribution Networks:

$$G(x, y, z) = \Phi_a(x, y)\Phi_b(x, y, z)\Phi_c(y, z)\Phi_d(z)$$



Thank you for listening!

A excellent book to start with:

S.Coles, *An Introduction to Statistical Modeling of Extreme Values*, 2001