Relative influence of mechanical and meteorological factors on avalanche release depth distributions:

An application to French Alps.

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EPFL – 12/06/2012

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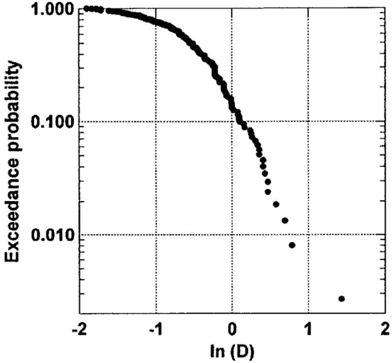
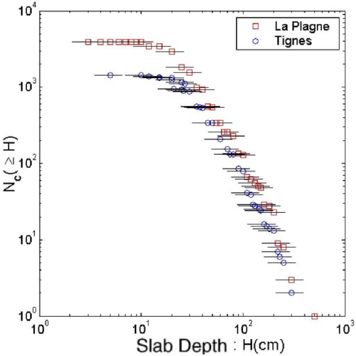
Context

Slab depth field measurements: Exceedence probability





- British Columbia (McClung, 2003)
- Tignes, La Plagne (Failletaz, 2006)

- Distribution tail = universal power law ?

Context

Release depth: Why is it important?

- Input ingredient of avalanche propagation models
- Objectives: risk management, zoning
  - hazard maps, Risk Prevention Plans (PPR),...
  - design and dimensioning of protection structures

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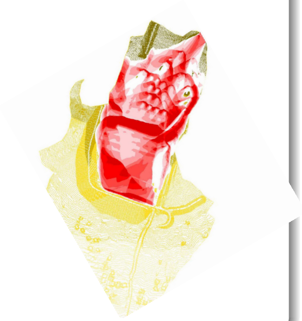

Traditional engineering assumption

The release depth corresponds to fresh snow accumulation (1 or 3 days). Is that true?

Theoretical framework: Assumptions and definitions

Avalanche occurence

- h: critical depth corresponding to a mechanical stability criterion.
- h_{sf}: snowfall depth.
- Natural release $\Leftrightarrow h_{sf} \geq h$

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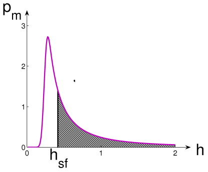
Definitions

- $p_m(h)$: Probability density of the mechanical critical depth h
- $p_{sf}(h_{sf})$: Probability density of the snowfall h_{sf}

Theoretical framework: Coupling equation



- $p(h|h_{sf}) = \begin{cases} \dfrac{p_m(h)}{\int_0^{h_{sf}} p_m(h')dh'} \text{ if } h \leq h_{sf} \\ 0 \text{ if } h \geq h_{sf} \end{cases}$
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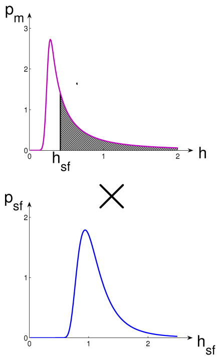

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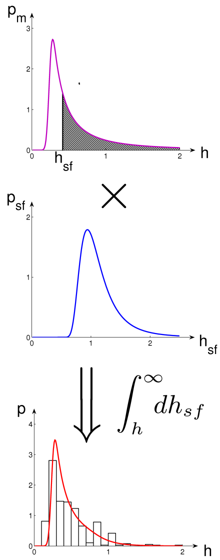

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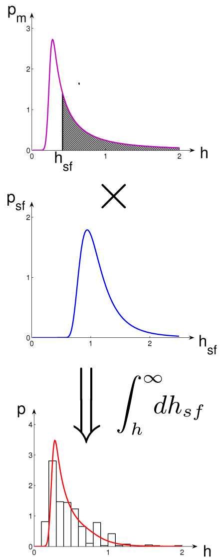

Mechanical probability density $p_m(h)$

$$p(h) = p_m(h) \int_h^{\infty} \frac{p_{sf}(h_{sf})}{\int_0^{h_{sf}} p_m(h')dh'} \, dh_{sf}$$

Slab avalanche release characteristics

Slab Avalanches

Slab avalanches generally result from the rupture of a weak layer underlaying a cohesive slab.

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Mechanical model for slab avalanche release

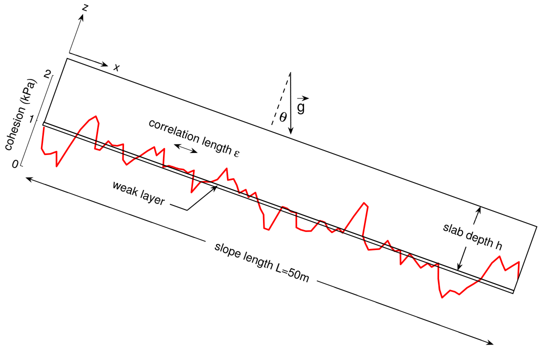
Objective

Build a mechanical model taking into account the following essential ingredients:

- slab - weak layer system
- the heterogeneity of weak-layer mechanical properties
- the redistribution of stresses by elasticity of the upperlaying slab

$\rightarrow$ release depth distributions.
$\rightarrow$ Finite Element Calculation using the FEM code Cast3m (CEA)

Simulated system



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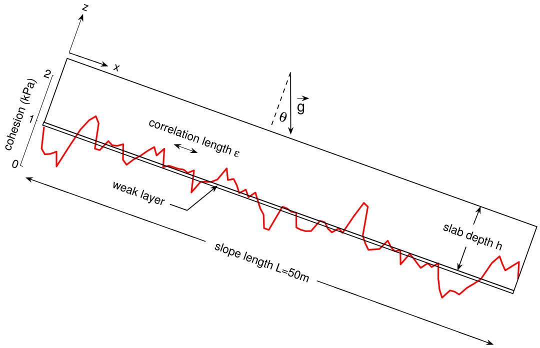


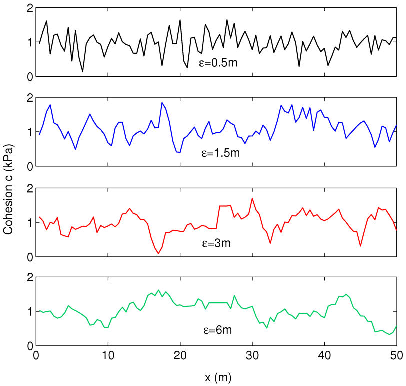
Constitutive laws

- **Weak layer**: Quasi-brittle (strain softening) interface law with a Mohr-Coulomb rupture criterion (cohesion c heterogeneous, friction angle $\phi = 30°$.)
- **Slab**: Elastic ($\rho = 250$ kg/m^3, $\nu = 0.2$, $E = 1$ MPa)

Weak-layer heterogeneity

- Weak layer cohesion c is modeled as a **Gaussian stochastic distribution with spatial correlations**:
  $c = N(\langle c \rangle, \sigma_c)$,
  with $CV = \frac{\sigma_c}{\langle c \rangle} \in 20 - 50\%$.
- **Spherical covariance**:
  $$C(d) = \sigma_c^2 \left(\frac{3}{2}\frac{d}{\epsilon} - \frac{1}{2}\left(\frac{d}{\epsilon}\right)^3 \right),$$
  with $\epsilon \in 0.5 - 10$ m.

Cohesion realizations
with $\langle c \rangle = 1$ kPa and $\sigma_c = 0.3$ kPa



Simulation protocol

$(\theta, \epsilon) \rightarrow$ 100 simulations with different realizations of cohesion heterogeneity c

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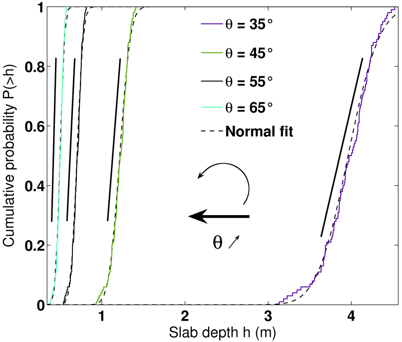
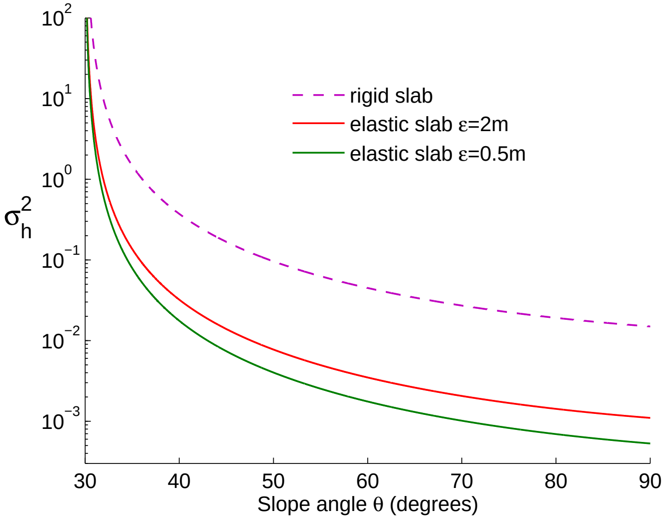
Result of the simulations

$$p(h|\theta) = \frac{1}{\sigma_h \sqrt{2\pi}} \, e^{-\frac{1}{2}\left(\frac{h - \langle h \rangle}{\sigma_h}\right)^2}$$

- $\langle h \rangle = \langle c \rangle / (\rho g F)$
- $\sigma_h = \sigma_c f(\epsilon) / (\rho g F)$
- $F = \sin\theta - \mu \cos\theta$
- $f(\epsilon) \approx \kappa (\epsilon/\Lambda)^{1/3}$ ($\kappa \approx 0.23$, $\Lambda \approx 1$ m)



Heterogeneity smoothing effect



Integration over all slope angles

$$p_m(h) = \int_{\theta_{min}}^{\theta_{max}} p(h|\theta)p(\theta)\,\mathrm{d}\theta$$

Assumption: $p(\theta)$ uniform between 30° and 90°

$$\Downarrow$$

$$p_m(h)$$

$$=$$

$$\frac{\sigma_c f(\epsilon)}{\rho g h^2 \sqrt{2\pi}} \left\{ e^{-\frac{1}{2}{U_1}^2} - e^{-\frac{1}{2}U_2(h)^2} + \sqrt{\frac{\pi}{2}} U_1 \left[\mathrm{erf}\left(\frac{U_1}{\sqrt{2}}\right) + \mathrm{erf}\left(\frac{U_2(h)}{\sqrt{2}}\right) \right] \right\}$$

- $U_1 = \langle c \rangle / [\sigma_c f(\epsilon)]$
- $U_2(h) = (\rho g h - \langle c \rangle) / [\sigma_c f(\epsilon)]$

Snowfall probability density $p_{sf}(h_{sf})$

$$p(h) = p_m(h) \int_h^\infty \frac{p_{sf}(h_{sf})}{\int_0^{h_{sf}} p_m(h')dh'}\, dh_{sf}$$

Which snowfalls?

- Avalanches occur during or after intense snowfalls $\Rightarrow$ Rare event $\Rightarrow$ Extreme snowfall analysis (GEV distribution)
- 3-day snowfall annual maxima = best avalanche predictor (Ancey&al 2003, Schweizer&al 2009)

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Extreme snowfall exceedance probability

- GEV distribution

$$p_{sf}(h_{sf} \geq h) = 1 - exp\left[-\left(1 + \xi \frac{h-\mu}{\sigma}\right)^{-1/\xi}\right]$$

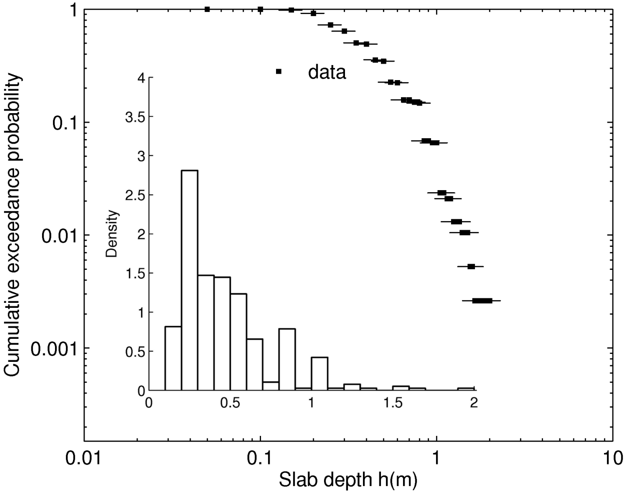
- Application to La Plagne snowfall data (MeteoFrance data: daily measurements since 1966):
$\mu = 0.98$m, $\sigma = 0.21$m and $\xi = 0.214$

Global probability density $p(h)$

$$p(h) = p_m(h) \int_h^{\infty} \frac{p_{sf}(h_{sf})}{\int_0^{h_{sf}} p_m(h')dh'} \, dh_{sf}$$

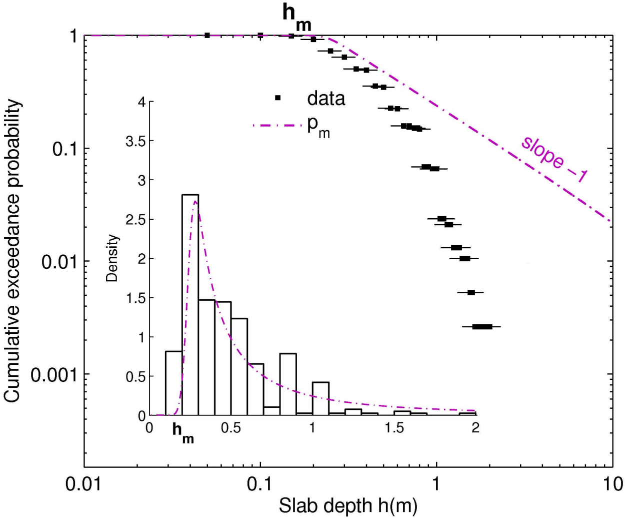
Global release depth distributions: La Plagne data

La Plagne release depth data: 369 natural slab avalanches



Global release depth distributions: La Plagne data

Mechanical probability density $p_m(h)$



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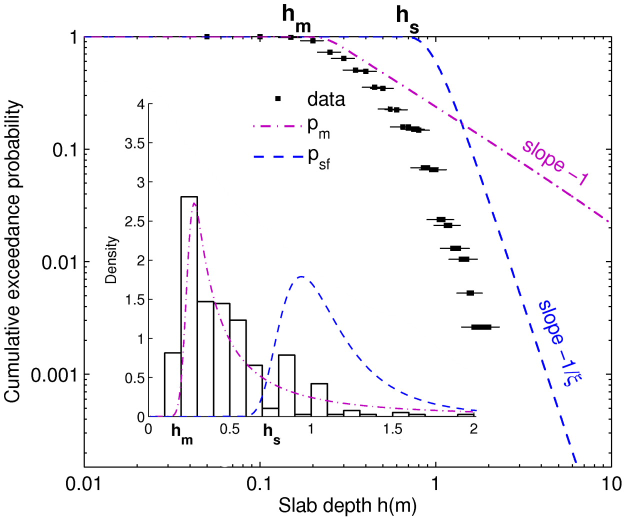

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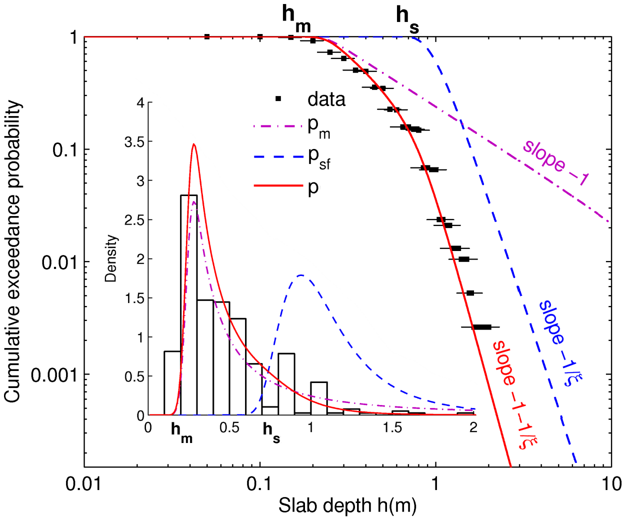
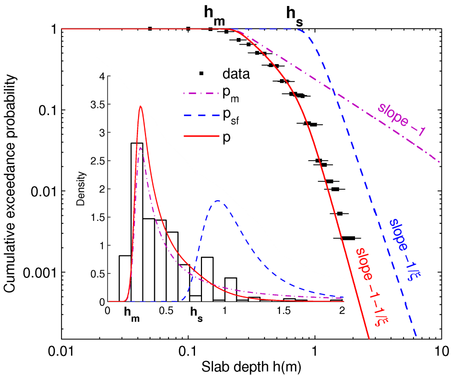


Result of the coupling for $\langle c \rangle = 0.6\text{kPa}$, $\sigma_c = 0.3\text{kPa}$, $\epsilon = 2\text{m}$



- adjustable parameter: mechanical cutoff

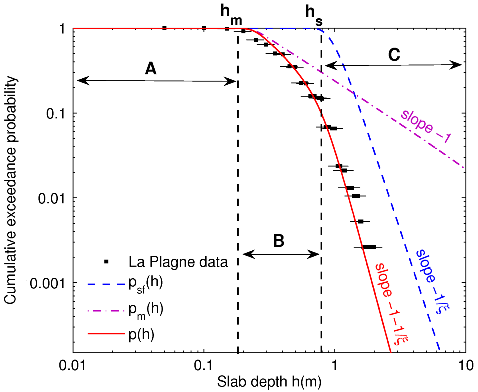
$$h_m \approx [\langle c \rangle - 2\sigma_c f(\epsilon)]/(\rho g)$$

- power-law slope:

$$\Psi = -1 - 1/\xi$$

- no universality of the release depth distributions

Global release depth distributions: Relative influence of mechanical and meteorological factors



Three zones:

- A: $h < h_m$, no avalanche
- B: $h_m \le h \le h_s$, **weak coupling** (mechanical effects are preponderant since snowfall are always sufficient)
- C: $h \ge h_s$, **strong coupling** (snowfalls become rarer and play the role of a limiting factor)

Slab depths predicted for a given return period are **lower** than with the classical engineering approach

How to predict the release depth where no data is available?

$\Downarrow$

- Spatialization of the snowfall data
- Spatialization of the average cohesion $\langle c \rangle$

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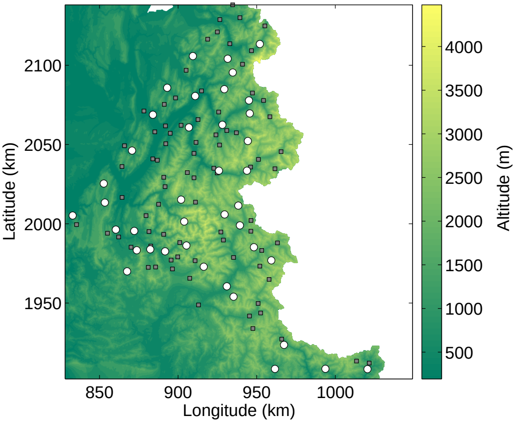
Data

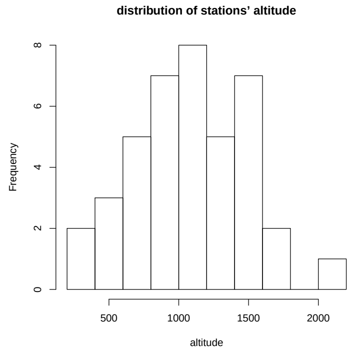


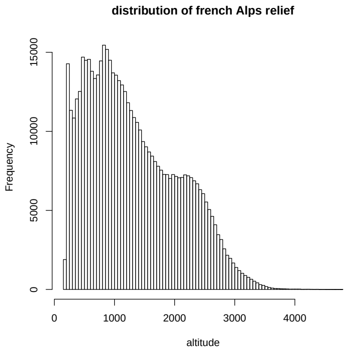
- $K = 40$ sites
- $N = 44$ years of mesurement (from 1966 to 2009)
- Daily measurements of snowfall (in mm w.eq)
- Data source: **MétéoFrance** Clim network.

Problematic

Weather station elevation

Weather stations are usually located at low altitude.



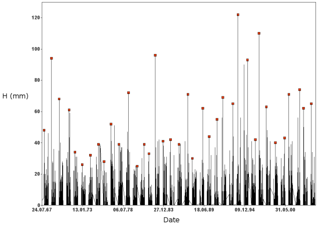


→ Need to take into account orographic gradient of altitude.

Problematic

Spatial interpolation of extreme values

Avalanches are rare events
$\rightarrow$ Analysis of extreme snowfalls



$\Rightarrow$ Need for spatial interpolation techniques specific to extreme values $\neq$ means
$\neq$ regional homogeneity (Salm et al 1990; Bocchiola et al 2006)
$\neq$ quantile smoothing (Weisse et Bois 2002)

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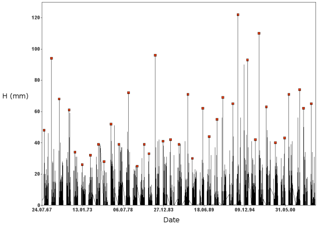

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Max-Stable Processes

New well established theoretical framework adapted for the spatial interpolation of extreme value.

Definition of a Max-Stable Process (de Haan, 1984)

$$H(x) = \lim_{n\to\infty} \frac{max_{i=1}^{n} Y_i(x) - b_n(x)}{a_n(x)},$$

$Y_i(x)_{x\in\mathbb{R}^d}$, n independent realisations of a continuous stochastic process, $a_n(x) > 0$ et $b_n(x) > 0$ two sequences of continuous functions.

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Consequences in the monovariate case

$H \equiv GEV(\mu, \sigma, \xi)$ with the following distribution function:

$$F(h;\mu,\sigma,\xi) = exp\left(-\left(1+\frac{\xi(h-\mu)}{\sigma}\right)_+^{-1/\xi}\right)$$

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Consequences in the spatial case (multivariate)

- Infinity of Max-Stable processes difficult to obtain and to use.
- The most known and used MS processes are **Smith ("Gaussian storm" model)**, Schlather and Brown-Resnick.

Spatial dependance: extremal coefficient

Definition

Notion of variogram or spatial correlation applied to extremes.

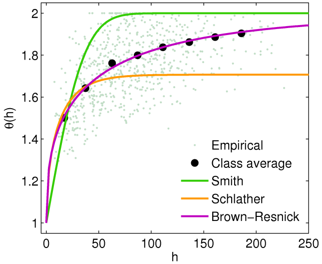
If $H(x)$ is a **Max-Stable process**

$$\Downarrow$$

$$\boxed{P(H(x) \leq u \text{ and } H(x+h) \leq u) = P(H(x) \leq u)^{\theta(h)}}$$

$\theta(h)$: extremal coefficient

$\theta = 1 \Rightarrow$ Perfect dependence
$\theta = 2 \Rightarrow$ Total independence



Spatial evolution models for the GEV parameters

Description of GEV parameters (μ, σ, ξ) through regression models or cubic splines which can be function of space (altitude, longitude, latitude), environment (orientation of the path, slope angle...) or random effects.
$\eta(x) = BX(x)$

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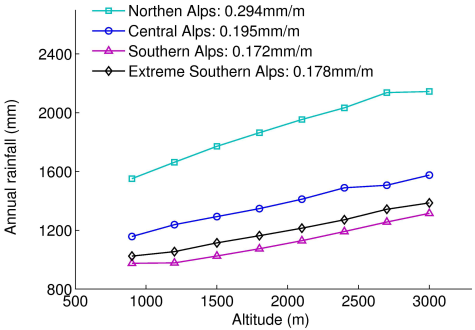
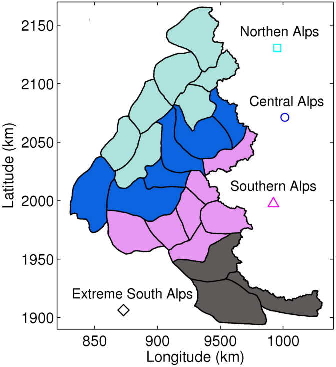
Example of spatial evolution

$$\begin{pmatrix} \mu \\ \sigma \\ \xi \end{pmatrix} = \begin{pmatrix} \beta_{\mu 0} & \beta_{\mu 1} & \beta_{\mu 2} \\ \beta_{\sigma 0} & \beta_{\sigma 1} & \beta_{\sigma 2} \\ \beta_{\xi 0} & \beta_{\xi 1} & \beta_{\xi 2} \end{pmatrix} \begin{pmatrix} 1 \\ \text{Lat} \\ \text{Long} \end{pmatrix}$$

Orographic gradient of altitude: Data transformation $\rightarrow$ 2000m (Durand et al 2009)





$$H_{2000}(x) = H_e(x) + \gamma(x)\frac{H_e(x)}{\mathrm{WMS}(x)}(2000 - e(x))$$

WMS: annual accumulation

Likelihood: Definition in the context of the pairwise analysis of spatial extremes (Padoan et al 2009)

$$l_c(\beta, H) = \sum_{n=1}^{N} \sum_{i=1}^{K} \sum_{j=i+1}^{K-1} \log f(H_{n,i}, H_{n,j}; \beta)$$

- K: number of sites
- N: number of years
- $i \in [1, ..., K]$
- $j \in [i+1, ..., K-1]$
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f: bivariate density of the model (Smith, Schlather ou Brown-Resnick)

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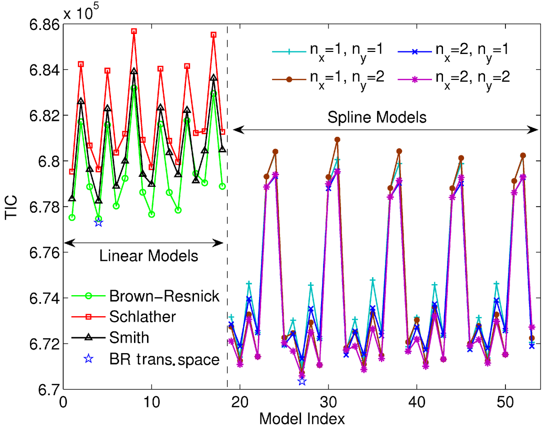
Estimation: Likelihood maximisation (MLE)

$$D_\beta l_c(\hat{\beta}_{MLE}, H) = 0 \rightarrow \hat{\beta}_{MLE}$$

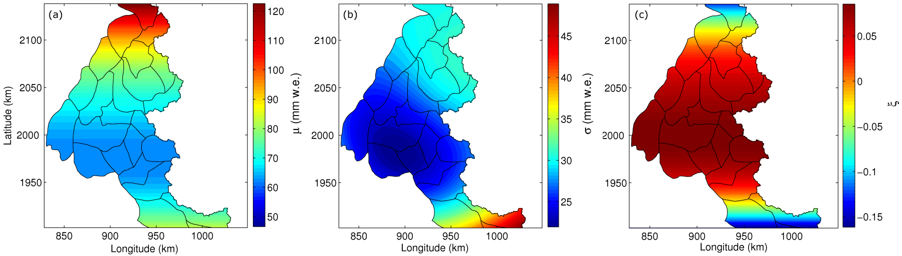
TIC Criterion (Takeuchi information criterion) for model selection

Best model: **Lower TIC value** (Takeuchi 1976)

$$\mathrm{TIC} \approx -2 l_c(\hat{\beta}_{MLE}, y) + 2p$$

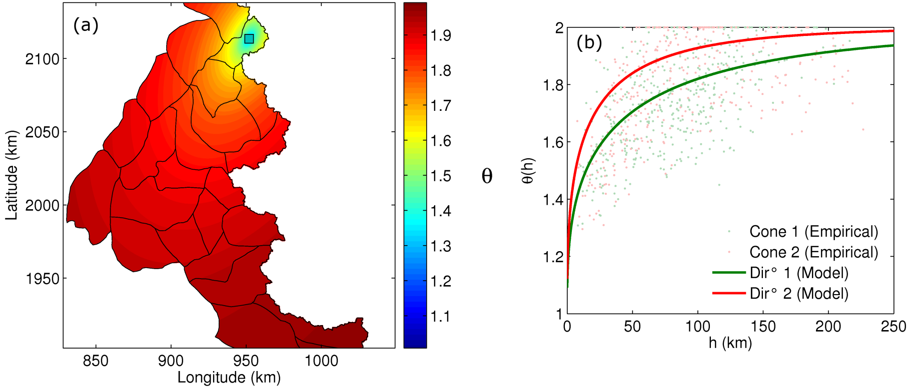


Spatial evolution of μ and σ and ξ



- Highest "means": North
- Highest "variances": South-East (Mediterranean effect)
- ξ mainly positive (Frechet domain) and negative (Weibull domain) in the extreme North and South.

Spatial dependence: Extremal coefficient $\theta(h)$



$$l_1 \approx 85 \text{ km}, \; l_2 \approx 185 \text{ km } (\theta = 1.9)$$
$$\alpha = 62.5^\circ$$

Quantile

Quantile calculation y_T

$$P(H \leq h_T) = \exp\left[-\left(1 + \frac{\xi(h_T - \mu)}{\sigma}\right)_+^{-1/\xi}\right] = 1 - \frac{1}{T}$$

$$\Rightarrow \boxed{h_T = \mu + \frac{\sigma}{\xi}\left[\left(-\ln\left(1 - \frac{1}{T}\right)\right)^{-\xi} - 1\right]}$$

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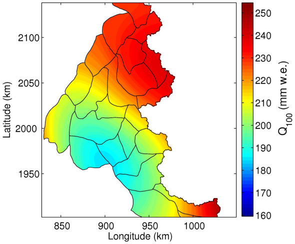
Calculation of the error on the quantile at 2000m

$$\Delta h_T = \Delta\mu + \left|\frac{1}{\xi}\left(\eta^{-\xi} - 1\right)\right|\Delta\sigma + \left|\frac{\sigma}{\xi}\left(\frac{1}{\xi}\left(\eta^{-\xi} - 1\right) + \eta^{-\xi}\ln\eta\right)\right|\Delta\xi$$

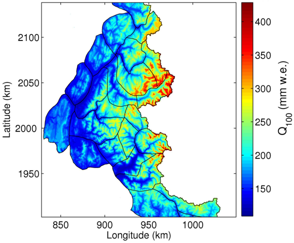
Avec $\eta = -\ln\left(1 - \frac{1}{T}\right)$

Quantile

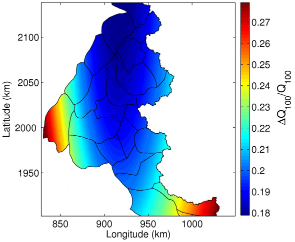
T =100ans



100-year quantile at 2000 m

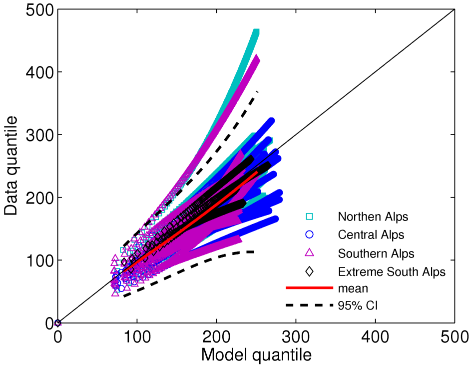


100-year quantile at real altitude

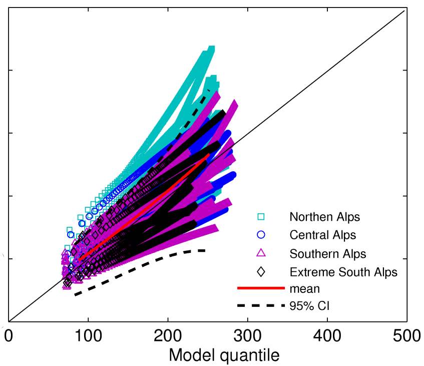


100-year quantile standard error

Validation



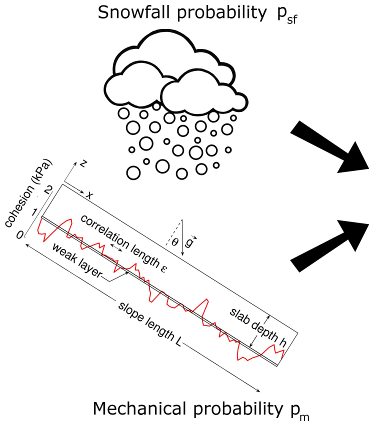
Local-spatial comparison at 2000 m

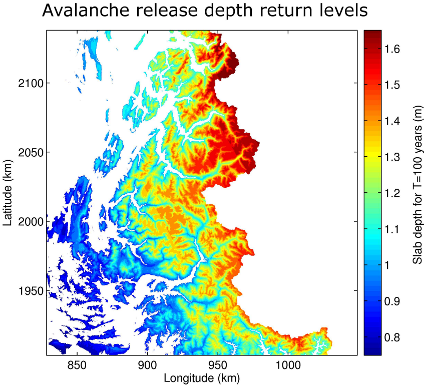


Cross validation for non-used stations at real altitude

Coupling using the mechanical-meteorological model

Spatialization using a max-stable model





Perspectives/Current developments

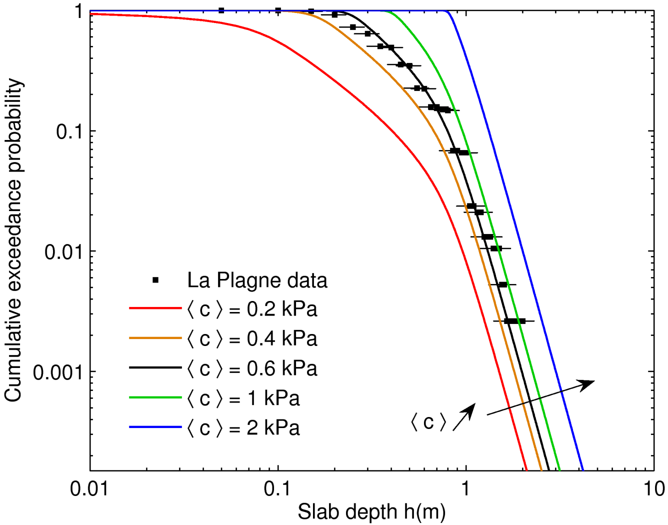
- Spatialization of the mechanical parameters (cohesion)
- Graphical User Interface

Highlights / Conclusions

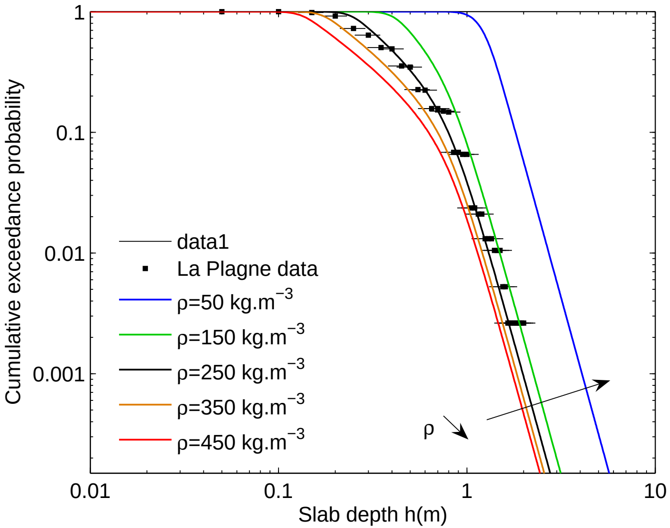
- Theoretical framework for the evaluation of avalanche release depth taking into account mechanical and meteorological effects
- Good agreement with field data
- Demonstrates the non-universality of these distributions
- Use of Max-Stable Processes for the spatialization of the extreme snowfall probability
- Enables to predict release depth distributions anywhere

Thanks for your attention

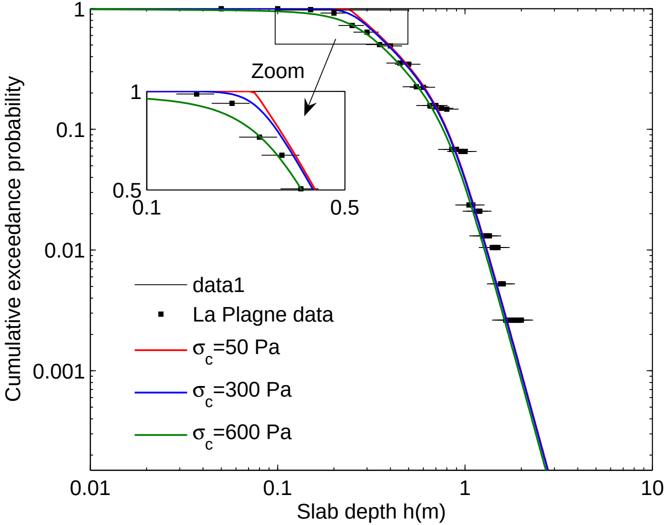
Appendix 1: Cohesion influence



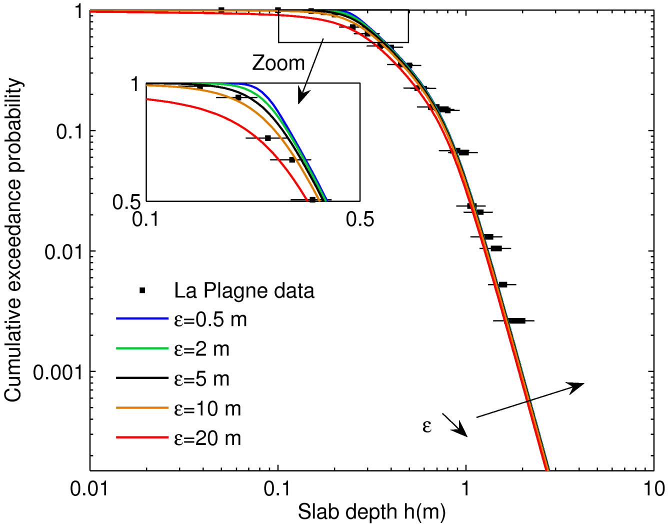
Appendix 2: Density influence



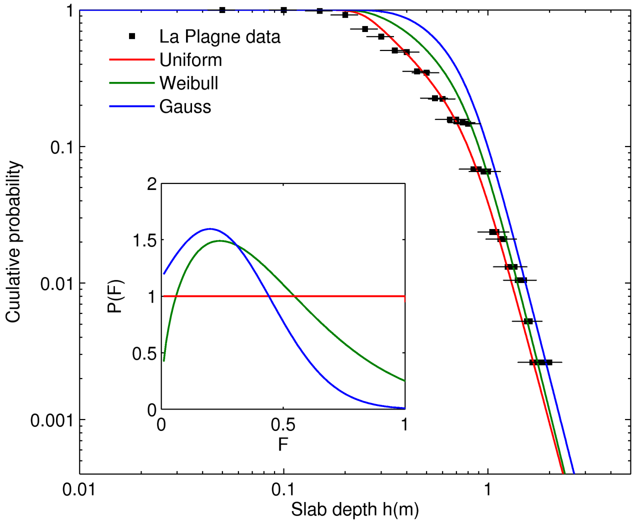
Appendix 3: Standard deviation influence



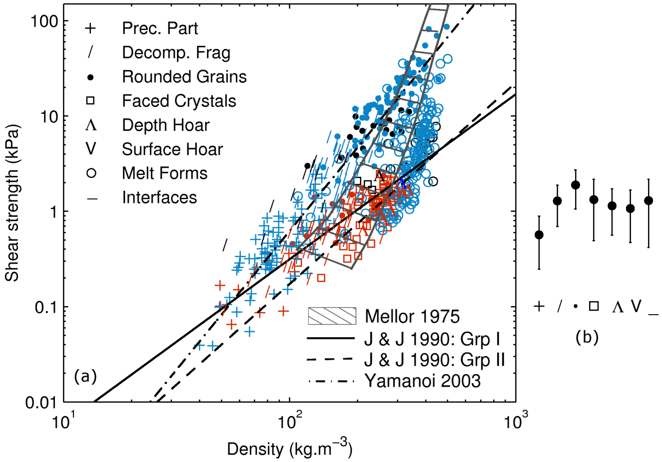
Appendix 4: Correlation length influence



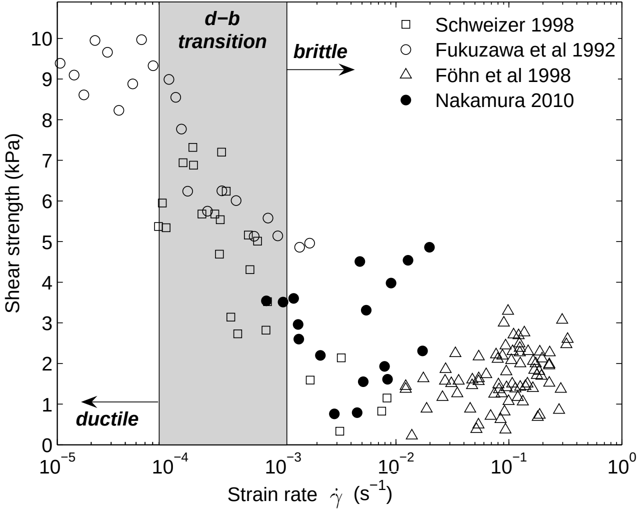
Appendix 5: Slope angle distribution influence



Appendix 6: Mechanical parameters: Shear strength



Appendix 7: Mechanical parameters: Brittle - ductile transition



Appendix 8: Mechanical parameters: Young modulus