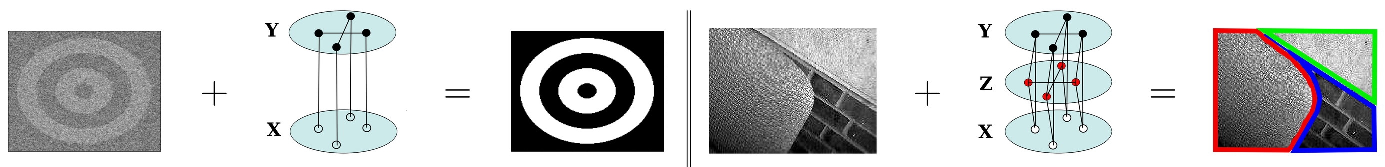
# Triplet Markov Field designed for supervised classification of texture images.

Juliette BLANCHET, Florence Forbes and Cordelia SCHMID
http://mistis.inrialpes.fr/people/blanchet
juliette.blanchet@inrialpes.fr

## 1. The problem and his modelization

- New probabilistic framework for supervised texture recognition in images.
- Statistical parametric model which takes into account both the spatial dependencies between observations and the non homogeneity of textures.




Observation $\mathbf{X}$
Hidden class (homogen) $\mathbf{Y}$

$(\mathbf{X},\mathbf{Y})$ HMF-IN
(Hidden Markov Field Independent Noise)

Observation $\mathbf{X}$
Hidden texture (unhomogen) $\mathbf{Y}$
+ Auxiliary variable $\mathbf{Z}$ ($\sim$ subclasses)

$(\mathbf{X},(\mathbf{Y},\mathbf{Z}))$ HMF-IN [recognizing step]
and $(\mathbf{X},\mathbf{Z}|\mathbf{y})$ HMF-IN [learning step]

## 2. The corresponding distributions

- $\forall i \in S$, observation $x_i \in \mathbb{R}^p$, hidden texture $y_i \in [1,L]$, auxiliary variable $z_i \in [1,K]$.

$$P(\mathbf{Y}=\mathbf{y}) = \frac{1}{W}\exp(\underbrace{\beta\sum_{i\sim j} 1_{y_i=y_j}}_{\text{a texture is a block}})$$

$$P(\mathbf{Z}=\mathbf{z}|\mathbf{Y}=\mathbf{y}) = \frac{1}{W(\mathbf{y})}\exp(\sum_{i\sim j}\underbrace{B_{y_iy_j}(z_i,z_j)}_{\substack{\text{correlation term}\\ \text{between subclasses}}})$$

$$P(\mathbf{x}|\mathbf{y},\mathbf{z}) = \prod_{i\in S} P(x_i|y_i,z_i) = \underbrace{\prod_{i\in S} f_{\theta_{y_iz_i}}(x_i)}_{\text{Independent Noise}}$$

- $\mathbf{Y}$ and $\mathbf{Z}|\mathbf{y}$ are markovians, so that $(\mathbf{Y},\mathbf{Z})$ is markovian
- $(\mathbf{X},\mathbf{Z}|\mathbf{y})$ is an HMF-IN
- $(\mathbf{X},\mathbf{Y},\mathbf{Z})$ is a TMF-IN (Triplet Markov Field with Independent Noise) [5]
- BUT $(\mathbf{X},\mathbf{Y})$ is not necessarily an HMF-IN

## 3. Learning Step

- We dispose of learning data labelled with the correponding texture $\mathbf{y}$.
- We estimate parameters $\{B_{ll'}\}$ and $\{\theta_{lk}\}$ of the HMF-IN $(\mathbf{X},\mathbf{Z}|\mathbf{Y}=\mathbf{y})$.
- For Hidden Markov Fields, due to the dependence structure, the exact Expectation-Maximisation (EM) algorithm [2] is untractable.
- We use the Mean Field (MF) approximation [3] to derive a factorized model:

$$P_G(\mathbf{z}|\mathbf{y}) \simeq \prod_{i=1}^{n} P(z_i|\mathbf{y},\tilde{z}_j, j\in N_i), \quad \tilde{\mathbf{z}} \text{ constant field}$$

Iterative algorithm MF-EM [4] which, at each iteration, repeats the two steps:
(i) create the new constant field $\tilde{\mathbf{z}} = (\tilde{z}_i)_{i\in S}$,
(ii) apply the EM algorithm for the factorized model to get new estimators of parameters $\{B_{ll'}\}$ and $\{\theta_{lk}\}$.

## 4. Recognizing Step

- We dispose of estimators $\hat{\theta}_{lk}$ of the $L\times K$ distributions $f_{\theta_{lk}}$ and estimators $\hat{B}_{ll'}$ of the $L\times L$ symmetric correlation matrices.
- Spatial parameter $\beta$ of the MRF $\mathbf{Y}$ remains to be unknown.
- Goal: to classify each unlabelled data from a new image in one of the $L$ textures.
- We run the MF-EM algorithm over the HMF-IN $(\mathbf{X},(\mathbf{Y},\mathbf{Z}))$ with fixed parameters $\hat{\theta}_{lk}$ and $\hat{B}_{ll'}$ to get new estimator $\hat{\beta}$ of $\beta$.
- Final classification is then obtained by applying the Most Probable Marginals (MPM) [1] rule according which the site $i$ is classified in texture $\hat{l}_i$ so that:

$$\hat{l}_i = \arg\max_{l\in[1,L]} P(Y_i=l|\mathbf{x}) = \sum_{k=1}^{K} P(Y_i=l, Z_i=k|\mathbf{x}).$$

## 5. Experimental results

- $L$=7 different textures.
- Learning base: $7\times 10 = 70$ single texture images.

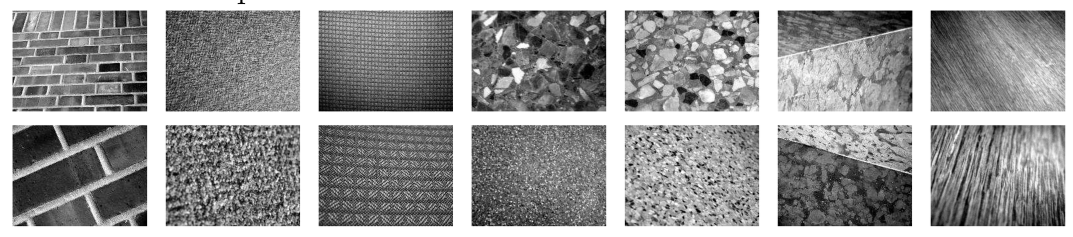



Figure 1: Sample of the learning base (7 textures).

- Recognition base: 70 single texture images and 68 multi-texture images.
- Observations are high-dimensional image descriptors (of dimension $p=128$) irregulary located.
- The neighborhood system is the Delaunay graph.

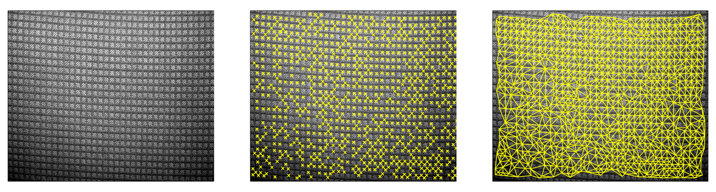


Figure 2: Original image, extracted descriptors and Delaunay neighborhood graph.

- Distributions $f_{\theta_{lk}}, l\in[1,L], k\in[1,K]$ ($L=7$, $K=10$) are supposed to be Gaussian, with diagonal covariance matrix ($\Sigma^{diag}$) or parametrized ($\Sigma^{hdim}$) [5] in order to cope with the dependencies between the 128 variables of a data.
- Independent Mixture model or Triplet Markov Field modelisation.

| Model | Covariance | Brick | Carpet | Chair | Floor 1 | Floor 2 | Marble | Wood |
|---|---|---|---|---|---|---|---|---|
| Ind. Mixture | $\Sigma^{diag}$ | 77.58 | 31.60 | 58.26 | 28.26 | 58.79 | 33.87 | 58.56 |
| Ind. Mixture | $\Sigma^{hdim}$ | 81.18 | 56.94 | 62.48 | 35.64 | 67.43 | 37.05 | 65.02 |
| Triplet MF | $\Sigma^{diag}$ | 96.59 | 80.70 | 83.60 | 82.69 | 83.90 | 46.05 | 95.18 |
| Triplet MF | $\Sigma^{hdim}$ | 99.33 | 98.61 | 99.28 | 97.36 | 99.57 | 56.24 | 99.28 |

Figure 3: Percent of data of each texture correctly classified.

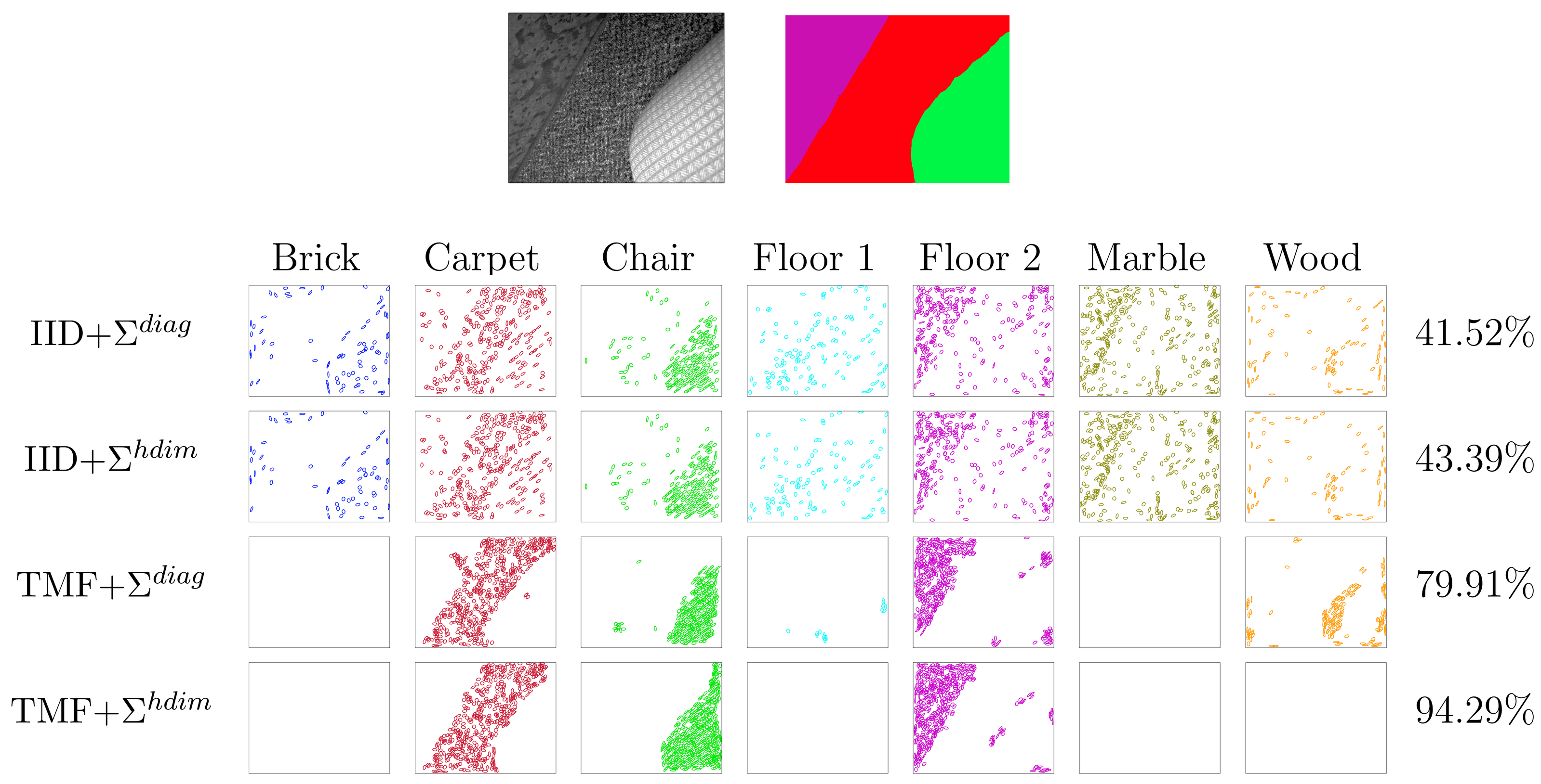



Figure 4: Classification of an image composed of 3 different textures (carpet, chair and floor 2) with, from top to bottom : independent mixture and $\Sigma^{diag}$, independent mixture and $\Sigma^{hdim}$, Triplet Markov Field and $\Sigma^{diag}$, Triplet Markov Field and $\Sigma^{hdim}$.

## Conclusion

- We have proposed a modelization designed for supervised classification of texture images based on Triplet Markov Field.
- Learning and classification steps are performed applying an EM-like algorithm with mean field approximation.
- Results obtained on real texture images are very satisfying.

## Bibliography

[1] J. Besag, *On the statistical analysis of dirty pictures.* Journal of the Royal Statistical Society, B 48(3), pp. 259–302, 1986.

[2] A. Dempster, N. Laird and D. Rubin, *Maximum Likelihood from incomplete data via the EM algorithm.* Journal of the Royal Statistical Society, 1977.

[3] D. Chandler, *Introduction to modern statistical mechanics.* Oxford University Press, 1987.

[4] G. Celeux, F. Forbes and N. Peyrard, *EM procedures using mean field-like approximations for Markov model-based image segmentation.* Pattern Recognition, 36(1) 131–144, 2003.

[5] C. Bouveyron, S. Girard and C. Schmid, *High Dimensional Discriminant Analysis.* Int. Conf. on Applied Stochastic Models and Data Analysis, pp. 526-534, Brest, France, 2005.

[6] D. Benboudjema and W. Pieczynski, *Unsupervised image segmentation using triplet Markov Fields.* Computer Vision and Image Understanding, pp. 476-498, 2005.