Lois de type Weibull

Soit $X_1, X_2, \ldots, X_n$ un ensemble de v.a. i.i.d. de fonction de répartition F telle que

$$1 - F(x) = \exp(-H(x)),\ H^{-1}(t) = t^{\theta}\ell(t),$$

où

- $\theta > 0$ est le coefficient de queue de type Weibull,
- ℓ est une fonction variations lentes *i.e.*

$$\ell(\lambda x)/\ell(x) \to 1 \text{ quand } x \to \infty \text{ pour tout } \lambda > 0.$$

H^{-1} est donc à variations régulières d'indice θ ce qui est noté $H^{-1} \in \mathcal{RV}_{\theta}$.

Exemples:

	θ	$\ell(x)$	$b(x)$	ρ
Loi normale $\mathcal{N}(\mu,\sigma^2)$	$1/2$	$2^{1/2}\sigma - \dfrac{\sigma}{2^{3/2}}\dfrac{\log x}{x} + O(1/x)$	$\dfrac{1}{4}\dfrac{\log x}{x}$	-1
Gamma $\Gamma(\alpha \neq 1, \beta)$	1	$\dfrac{1}{\beta} + \dfrac{\alpha-1}{\beta}\dfrac{\log x}{x} + O(1/x)$	$(1-\alpha)\dfrac{\log x}{x}$	-1
Weibull $\mathcal{W}(\alpha,\lambda)$	$1/\alpha$	λ	0	$-\infty$

Estimation du coefficient de queue de type Weibull:

Soit $q(t)$ la fonction quantile

$$q(t) = F^{-1}(1-t) = H^{-1}(\log(1/t)) = (\log(1/t))^{\theta}\ell(\log(1/t)).$$

Pour t et s petits:

$$\begin{aligned}\log(q(t)) - \log(q(s)) &= \theta(\log\log(1/t) - \log\log(1/s)) + \log\left(\frac{\ell(\log(1/t))}{\ell(\log(1/s))}\right) \\ &\simeq \theta(\log\log(1/t) - \log\log(1/s)),\end{aligned}$$

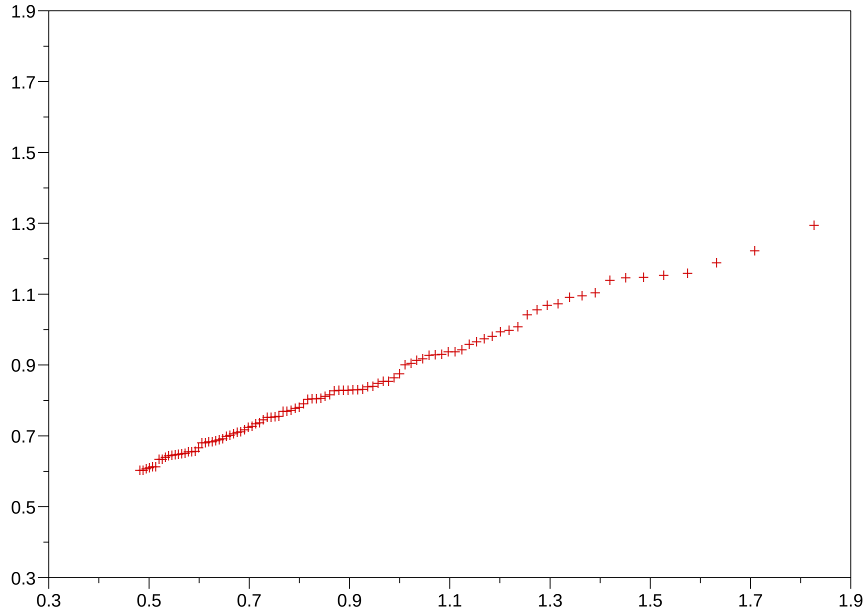
En considérant $t = i/n$, $s = k_n/n$ et en remplaçant F par la fonction de répartition empirique, on obtient

$$\log(X_{n-i+1,n}) - \log(X_{n-k_n+1,n}) \simeq \theta(\log\log(n/i) - \log\log(n/k_n)),$$

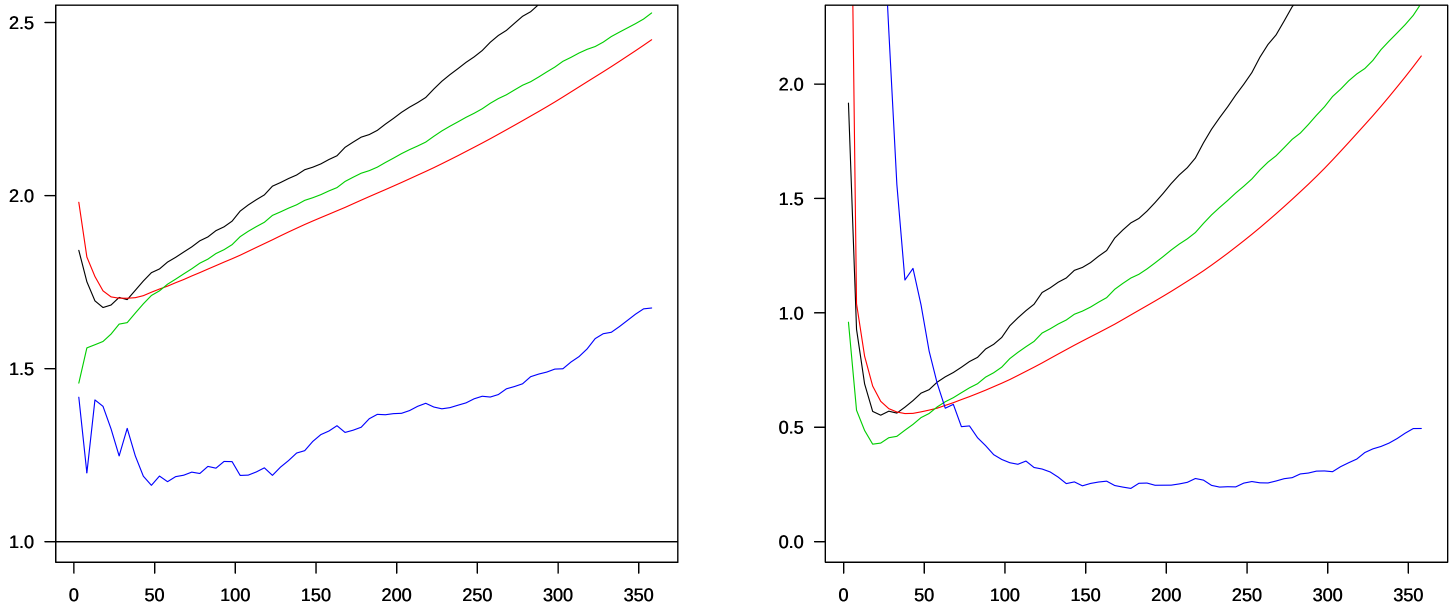
pour $i = 1, \ldots, k_n - 1$.

QQ-plot:

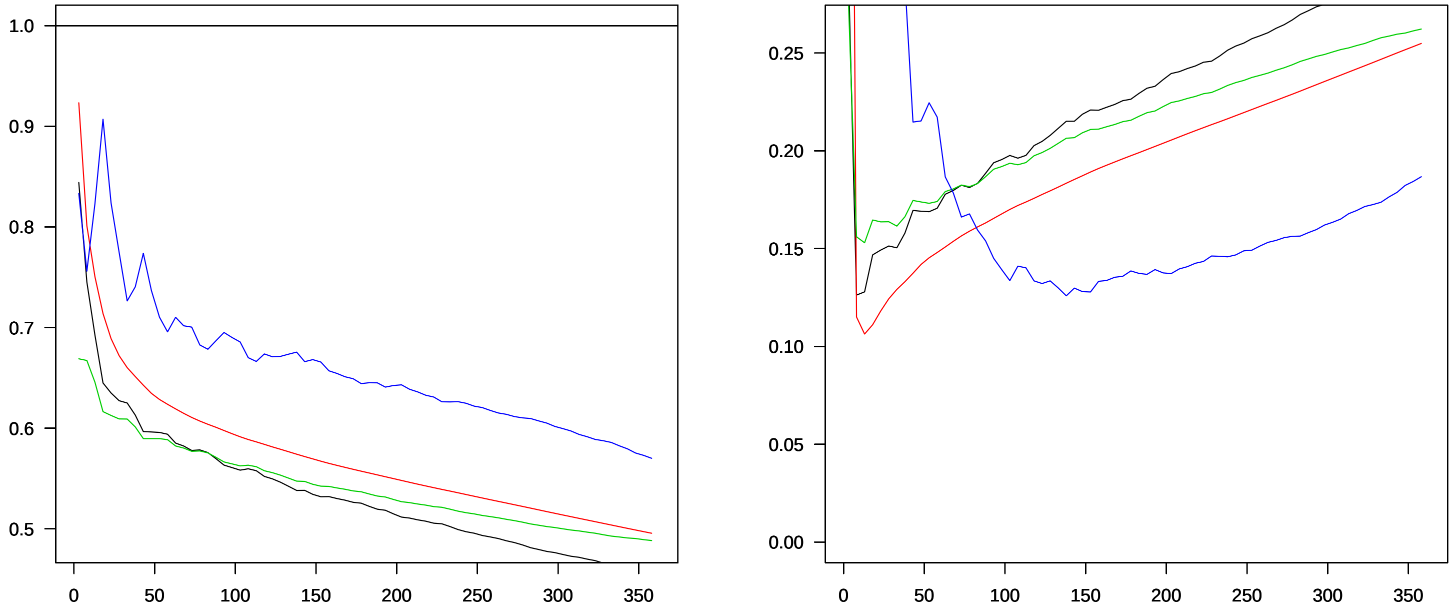
- Couples $(\log\log(n/i), \log(X_{n-i+1,n}))$ pour $i = 1, \dots, k_n$.
- Exemple : loi $\mathcal{N}(0,1)$ $n = 500$, $k_n = 100$.



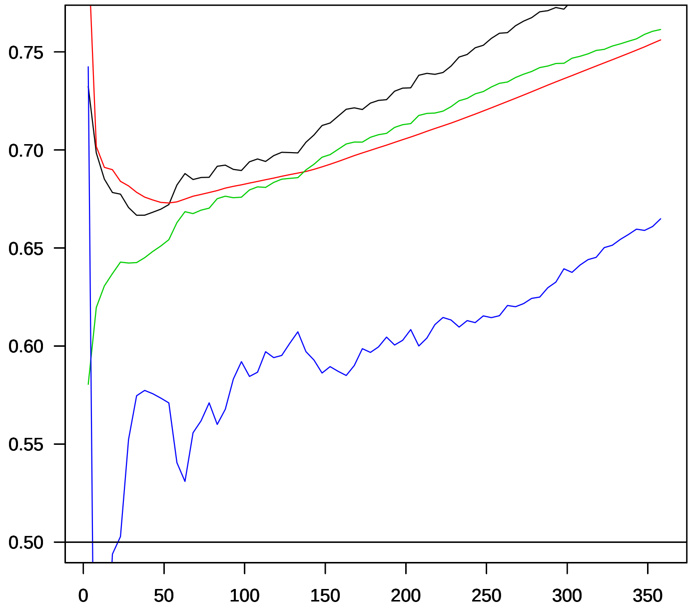
$$\text{Gamma } \Gamma(0.25, 1) \longrightarrow \theta = 1,\ b(x) > 0$$

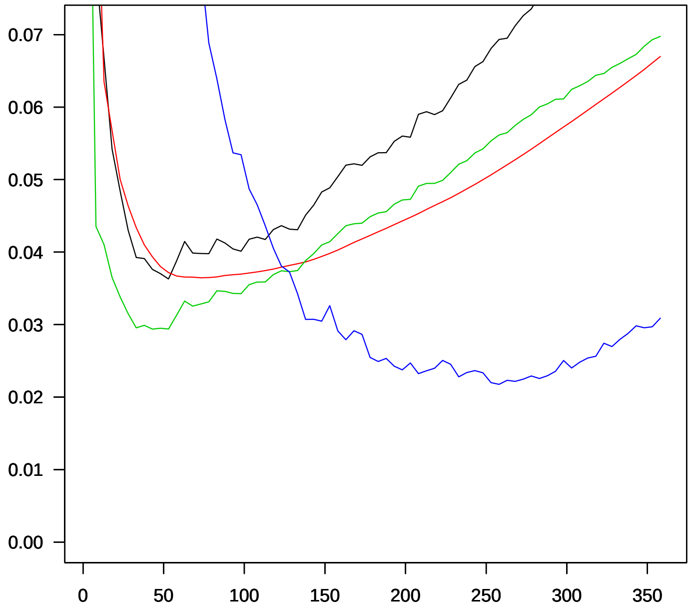


Gamma $\Gamma(4,1) \longrightarrow \theta = 1,\ b(x) < 0$

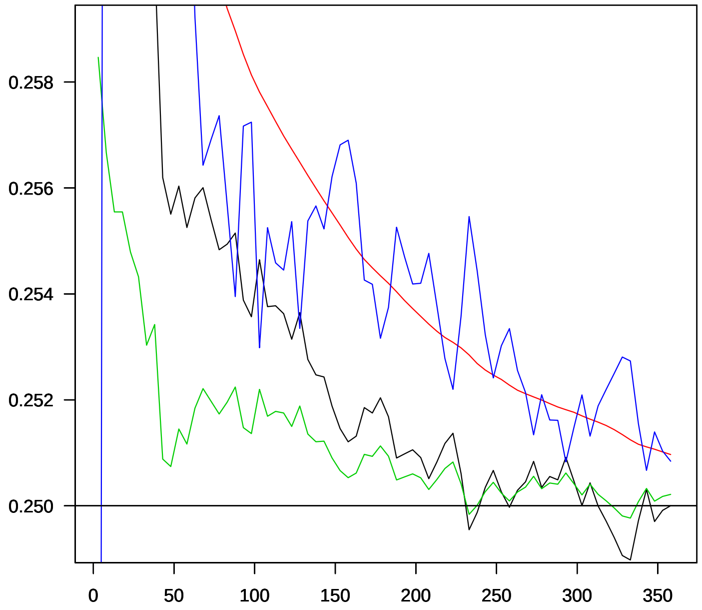


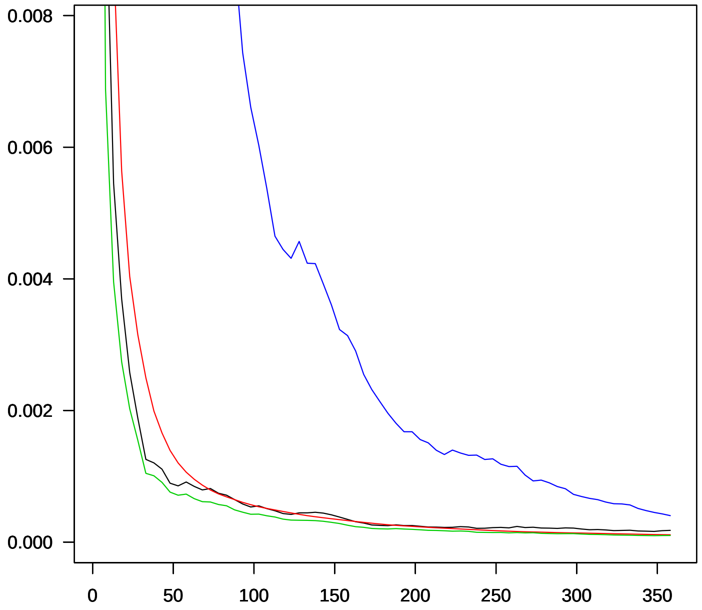
Loi normale $\mathcal{N}(0,1) \longrightarrow \theta = 1/2,\ b(x) > 0$



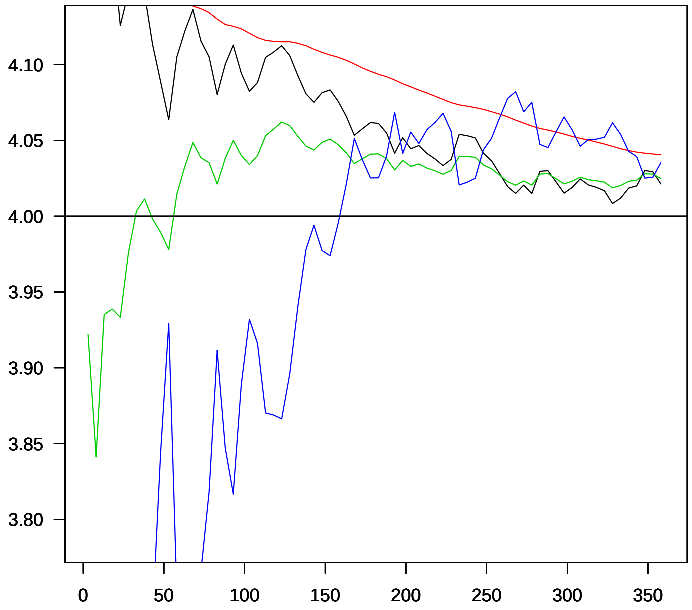


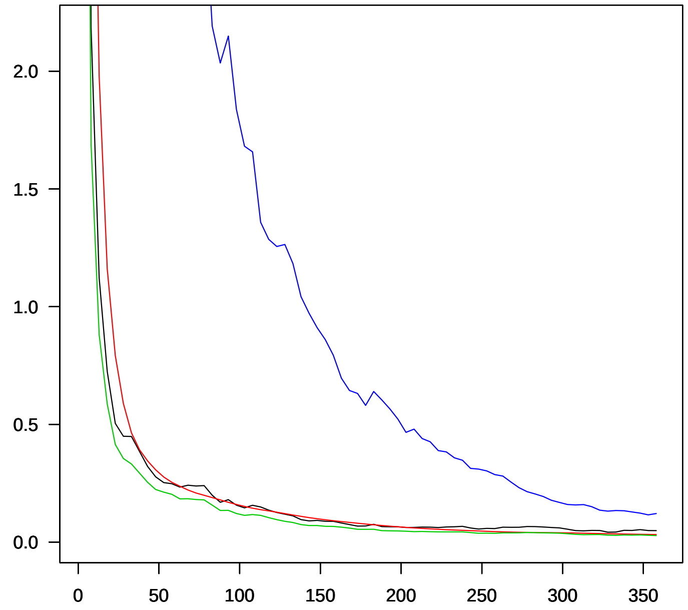
Weibull $\mathcal{W}(4,4) \longrightarrow \theta = 0.25,\ b(x) = 0$





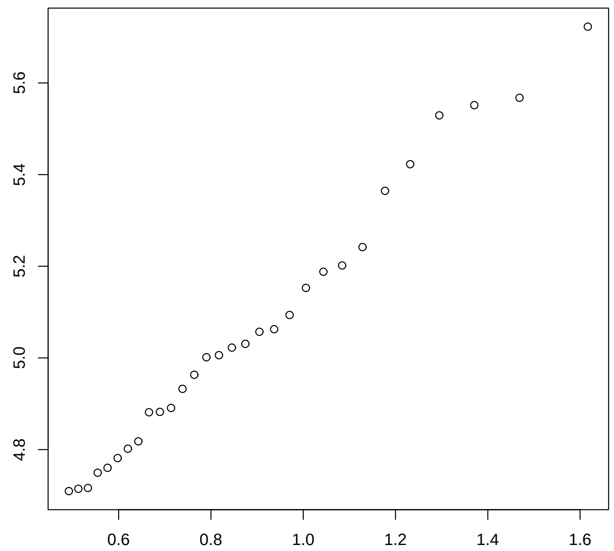
Weibull $\mathcal{W}(0.25, 0.25) \longrightarrow \theta = 4,\ b(x) = 0$





Rivière Nidd :

154 mesures de débit de la rivière Nidd (Yorkshire, England) durant la période 1934-1969.



QQ-plot

$k_n = 29$, $\hat{\theta}_n \simeq 0.89$, période de retour centennale: $\hat{x}_{p_n}(\hat{\theta}_n) = 366 m^3 s^{-1}$