Contributions to extreme-value analysis

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Abstract: This report summarizes my contributions to extreme-value statistics. I worked on the estimation of the extreme-value index, Weibull-tail coefficient and end-point estimation. I also studied conditional extremes, that is the situation where the extreme-value behavior depends on a covariate.

Contributions

Extreme value theory is a branch of statistics dealing with the extreme deviations from the bulk of probability distributions. More specifically, it focuses on the limiting distributions for the minimum or the maximum of a large collection of random observations from the same arbitrary (unknown) distribution. Let $x_1 < \cdots < x_n$ denote n ordered observations from a random variable X representing some quantity of interest. A p_n-quantile of X is the value q_{p_n} such that the probability that X is greater than q_{p_n} is p_n, i.e. $P(X > q_{p_n}) = p_n$. When $p_n < 1/n$, such a quantile is said to be extreme since it is usually greater than the maximum observation x_n. To estimate such extreme quantiles requires therefore specific methods to extrapolate information beyond the observed values of X. Those methods are based on Extreme value theory. This kind of issues appeared in hydrology. One objective was to assess risk for highly unusual events, such as 100-year floods, starting from flows measured over 50 years.

The decay of the survival function $P(X > x) = 1 - F(x)$, where F denotes the cumulative distribution function associated to X, is driven by a real parameter called the extreme-value index γ. I have proposed several estimators for this parameter, see [1, 2, 3, 4, 5]. When this parameter is positive, the survival function is said to be heavy-tailed, when this parameter is negative, the survival function vanishes above its

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right end point. Some estimation methods for the right end point have been proposed in [6, 7]. If this parameter is zero, then the survival function decreases to zero at an exponential rate. An important part of our work is dedicated to the study of such distributions. For instance, in reliability, the distributions of interest are included in a semi-parametric family whose tails are decreasing exponentially fast. These so-called Weibull tail-distributions encompass a variety of light-tailed distributions, such as Weibull, Gaussian, gamma and logistic distributions. Let us recall that a cumulative distribution function F has a Weibull tail if it satisfies the following property: There exists $\theta > 0$ such that for all $\lambda > 0$,

$$\lim_{y \to \infty} \frac{\log(1 - F(\lambda y))}{\log(1 - F(y))} = \lambda^{1/\theta}.$$

Dedicated methods have been proposed to estimate the Weibull tail-coefficient θ since the relevant information is only contained in the extreme upper part of the sample. More specifically, the estimators I proposed are based on the log-spacings between the upper order statistics [8, 9, 10, 11, 12]. See also [13, 14, 15, 16] for the estimation of the associated extreme quantiles. These methods can also be seen as an improvement of the ET method [17, 18, 19]. Of course, the choice of a tail model is an important issue, see [20] for the introduction of a goodness-of-fit test and [21] for its implementation.

I also addressed the estimation of extreme level curves. This problem is equivalent to estimating quantiles when covariate information is available and when their order converges to one as the sample size increases. We show that, under some conditions, these so-called "extreme conditional quantiles" can still be estimated through a kernel estimator of the conditional survival function. Sufficient conditions on the rate of convergence of their order to one are provided to obtain asymptotically Gaussian distributed estimators. Making use of this result, some estimators of the extreme-value parameters are introduced and extreme conditional quantiles estimators are deduced [22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. Finally, the tail copula is widely used to describe the dependence in the tail of multivariate distributions. In some situations such as risk management, the dependence structure may be linked with some covariate. The tail copula thus depends on this covariate and is referred to as the conditional tail copula. The aim of [32] is to propose a nonparametric estimator of the conditional tail copula and to establish its asymptotic normality.

In the multivariate context, we focus on extreme geometric quantiles [33, 34]. Their asymptotics are established, both in direction and magnitude, under suitable integrability conditions, when the norm of the associated index vector tends to one.

Applications of extreme-value theory are found in ecology [35], hydrology [36, 37, 38, 39] and more generally in risk estimation [40, 41, 42].

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