Inverting hyperspectral images with Gaussian Regularized Sliced Inverse Regression

Caroline Bernard-Michel, Sylvain Douté, Laurent Gardes and Stéphane Girard

MISTIS - INRIA Rhône-Alpes
http://mistis.inrialpes.fr/

The inverse problem

- Visible and near infrared imaging spectroscopy allows the detection, mapping and characterization of minerals and ices by analyzing the solar light reflected in different directions by the surface materials.
- Modeling the direct link between some physical parameters \( Y \) and observable spectra \( X \) is called the forward problem and allows, for given values of the model parameters, to simulate the spectra that should be observed.
- Conversely, deducing the physical model parameters from the observed spectra is called an inverse problem.
- Application to OMEGA/MEX hyperspectral images observed on Mars [2].

Functional approach and dimension reduction

- Estimate the functional relationship \( F \) between the spectra \( X \in \mathbb{R}^p \) and one parameter \( Y \in \mathbb{R} \) (\( p = 184 \) wavelengths).
- Because of the curse of dimensionality, dimension reduction techniques are required.
- They rely on the assumption that the predictor \( X \) can be replaced by its projection on a subspace of smaller dimension \( X' \) without loss of information. Denoting by \( \beta_1, \ldots, \beta_K \) a basis of this subspace, the functional relationship \( Y = f(X) \) can be rewritten as \( Y = f(\beta_1 X, \ldots, \beta_K X) \) where \( f \) is now a \( K \)-variate function.

Sliced Inverse regression

- Introduced by Li [4]
  - Find the directions \( h = (\beta_1, \ldots, \beta_K) \) such that \( h' X \) best explains \( Y \).
  - Find the directions \( h \) minimizing the variances of \( h' X \) given \( Y \).
- In practice, the range of \( Y \) is partitioned into \( h \) slices and one needs to calculate the eigenvectors of \( (\Sigma h)^{-1} \) where \( \Sigma \) is the spectra covariance matrix and \( h \) the slice mean spectra covariance matrix.

Our approach

- One GRSIR axis is sufficient.
- The \( k \)-nearest neighbors methodology (\( k \)-NN) is very unstable.
- GRGSIR gives the best results in terms of Normalized Root Mean Square Errors (NRMSE) for most parameters.
- PLS [3] does not seem suited because the relationship is non linear.
- There is still a small bias with GRSIR due to the choice of the learning database.

Inversion of real hyperspectral images

- Validation is difficult because no ground truth data is available.
- GRGSIR first axis does put weights on key spectral points according to researchers in planetary physics.
- GRGSIR estimations vary continuously and seem to be spatially coherent.
- GRGSIR map is more detailed.
- GRGSIR is in accordance with the Waveanglet physical approach whereas in some regions, \( k \)-NN and PLS give conflicting estimations. Waveanglet is a supervised classification method that allows the detection and quantification of major compounds on hyperspectral images [5].
- Images from different orbits but analyzing the same portion of Mars give similar GRGSIR estimates.
- When spectra cannot be inverted by GRSIR, it generally means they correspond to another physical model.

Bibliography