Contributions to extreme-value statistics for risk analysis

Stéphane Girard

INRIA Rhône-Alpes & LJK (team MISTIS).

655, avenue de l'Europe, Montbonnot. 38334 Saint-Ismier Cedex, France

Stephane.Girard@inria.fr

Extreme value theory is a branch of statistics dealing with the extreme deviations from the bulk of probability distributions. More specifically, it focuses on the limiting distributions for the minimum or the maximum of a large collection of random observations from the same arbitrary (unknown) distribution. Let $x_1 < \cdots < x_n$ denote n ordered observations from a random variable X representing some quantity of interest. A p_n-quantile of X is the value q_{p_n} such that the probability that X is greater than q_{p_n} is p_n, i.e. $P(X > q_{p_n}) = p_n$. When $p_n < 1/n$, such a quantile is said to be extreme since it is usually greater than the maximum observation x_n. To estimate such extreme quantiles requires therefore specific methods to extrapolate information beyond the observed values of X. Those methods are based on Extreme value theory. This kind of issues appeared in hydrology. One objective was to assess risk for highly unusual events, such as 100-year floods, starting from flows measured over 50 years. Such return levels are referred to as Value-at-Risk in finance and to extreme quantiles in extreme value statistics.

We introduced a new risk measure, the so-called Conditional Tail Moment. It is the moment of order $a > 0$ of the loss distribution above the upper α-quantile. Estimating the Conditional Tail Moment permits to estimate all risk measures based on conditional moments such as Conditional Tail Expectation [1], Conditional Value-at-Risk or Conditional Tail Variance. We focussed on the estimation of these risk measures in case of extreme losses (where α converges to 0). It is moreover assumed that the loss distribution depends on a covariate. The estimation method thus combines nonparametric kernel methods with extreme-value statistics [2, 3]. We also refer to [4] for the estimation of the proportional hazard premium. Quantiles can be embedded in a more general class of M-quantiles by means of L_p optimization [5]. These generalized L_p quantiles steer an advantageous middle course between ordinary quantiles and expectiles [6] without sacrificing their virtues too much for $p \in [1, 2]$. We investigated their estimation from the perspective of extreme values in the class of heavy-tailed distributions. We construct estimators of the intermediate L_p quantiles and establish their asymptotic normality in a dependence framework motivated by financial and actuarial applications, before extrapolating these estimates to the very far tails. We also investigate the potential of extreme L_p quantiles as a tool for estimating the usual quantiles and expectiles themselves.

In the multivariate context, we focus on extreme geometric quantiles [7, 8]. Their asymptotics are established, both in direction and magnitude, under suitable integrability conditions, when the norm of the associated index vector tends to one. In particular, it appears that if a random vector has a finite covariance matrix, then the magnitude of its extreme geometric quantiles grows at a fixed rate which is independent of the asymptotic behaviour of the underlying probability distribution. Moreover, in the special case of elliptically contoured distributions, the respective shapes of the contour plots of extreme geometric quantiles and extreme level sets of the probability density function are orthogonal, in some sense. These phenomena are illustrated on some numerical examples.

Applications are developed in ecology [9], hydrology [10, 11, 12, 13, 14].

References

[1] E. Deme, S. Girard, and A. Guillou. Reduced-bias estimators of the conditional tail expectation for heavy-tailed distributions. In M. Hallin et al., editor, *Mathematical Statistics and Limit Theorems*, pages 105–123. Springer, 2015.

[2] J. El Methni, L. Gardes, and S. Girard. Nonparametric estimation of extreme risks from conditional heavy-tailed distributions. *Scandinavian Journal of Statistics*, 41:988–1012, 2014.

[3] J. El Methni, L. Gardes, and S. Girard. Kernel estimation of extreme regression risk measures. *Electronic Journal of Statistics*, 2018. to appear.

[4] E. Deme, S. Girard, and A. Guillou. Reduced-bias estimator of the proportional hazard premium for heavy-tailed distributions. *Insurance: Mathematics and Economics*, 22:550–559, 2013.

[5] A. Daouia, S. Girard, and A. G. Stupfler. Extreme M-quantiles as risk measures: From L_1 to L_p optimization. *Bernoulli*, 2018. to appear.

[6] A. Daouia, S. Girard, and G. Stupfler. Estimation of tail risk based on extreme expectiles. *Journal of the Royal Statistical Society series B*, 80:263–292, 2018.

[7] S. Girard and G. Stupfler. Intriguing properties of extreme geometric quantiles. *REVSTAT - Statistical Journal*, 15:107–139, 2017.

[8] S. Girard and G. Stupfler. Extreme geometric quantiles in a multivariate regular variation framework. *Extremes*, 18(4):629–663, 2015.

[9] M. Stehlik, P. Aguirre, S. Girard, P. Jordanova, J. Kiselak, S. Torres, Z. Stadovsky, and A. Rivera. On ecosystems dynamics. *Ecological Complexity*, 29:10–29, 2017.

[10] B. Barroca, P. Bernardara, S. Girard, and G. Mazo. Considering hazard estimation uncertain in urban resilience strategies. In K. Etingoff, editor, *Ecological Resilience, Response to Climate Change and Natural Disasters*, pages 197–220. Apple Academic Press, 2016.

[11] J. El Methni, L. Gardes, and S. Girard. Estimation de mesures de risque pour des pluies extrêmes dans la région Cévennes Vivarais. *La Houille Blanche*, 4:46–51, 2015.

[12] J. Carreau, D. Ceresetti, E. Ursu, S. Anquetin, J.D. Creutin, L. Gardes, S. Girard, and G. Molinié. Evaluation of classical spatial-analysis schemes of extreme rainfall. *Natural Hazards and Earth System Sciences*, 12:3229–3240, 2012.

[13] B. Barroca, P. Bernardara, S. Girard, and G. Mazo. Considering hazard estimation uncertain in urban resilience strategies. *Natural Hazards and Earth System Sciences*, 15:25–34, 2015.

[14] L. Gardes and S. Girard. Conditional extremes from heavy-tailed distributions: An application to the estimation of extreme rainfall return levels. *Extremes*, 13(2):177–204, 2010.