Brain Lesion Segmentation: A Bayesian Weighted EM Approach

Senan Doyle, Florence Forbes, Michel Dojat

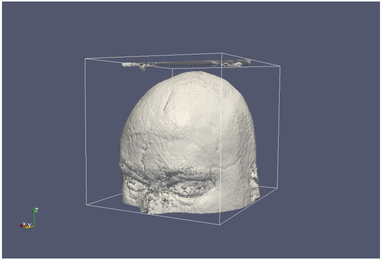
November 19, 2009

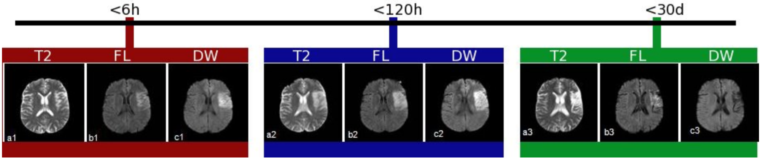
Table of contents

Goal

Delineation & Quantification of Lesions

- Establish Patient Prognosis
- Chart Development over Time





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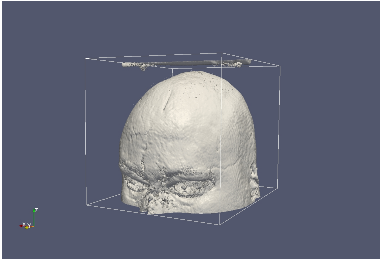

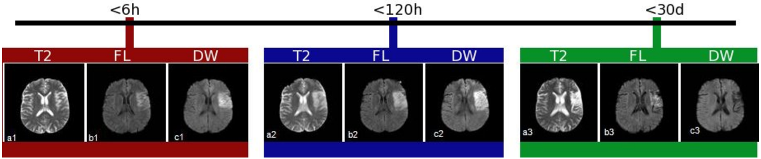

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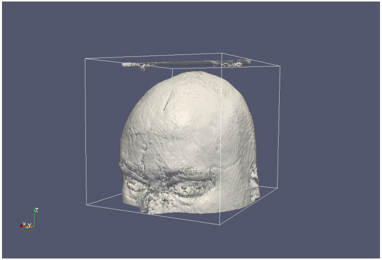

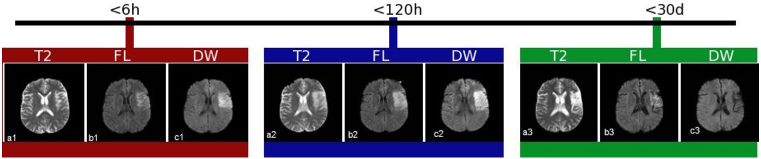

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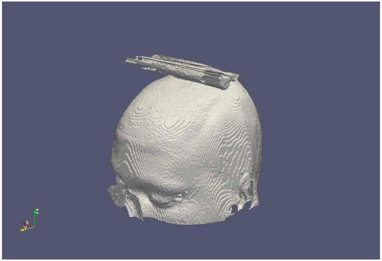

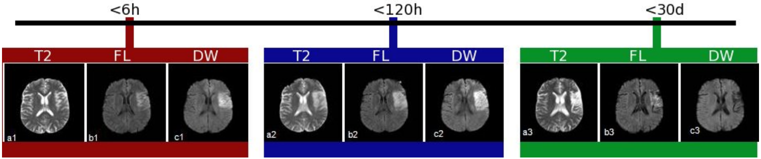

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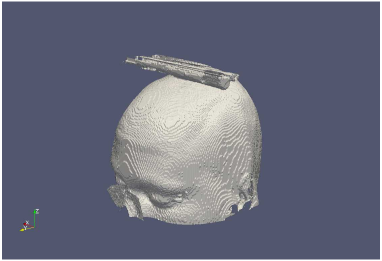

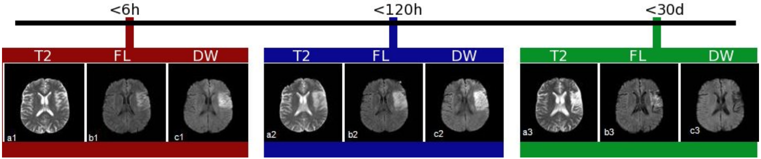

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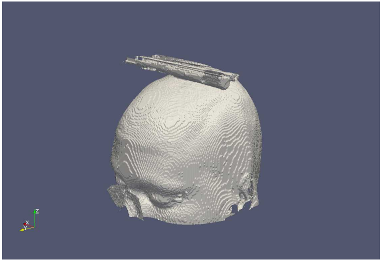

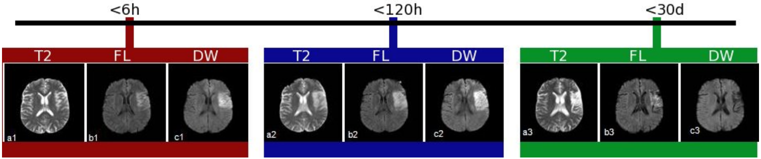

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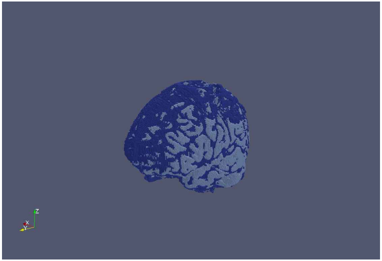

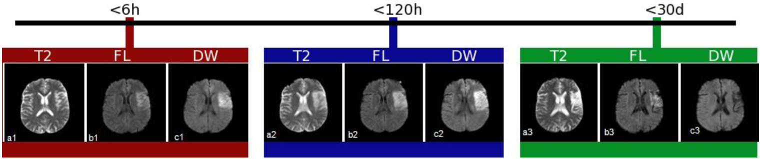

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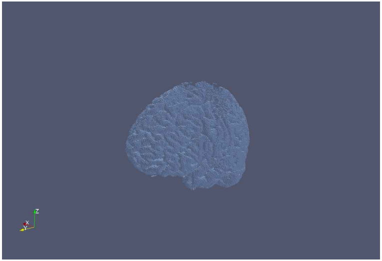

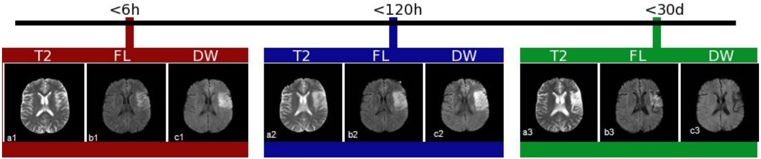

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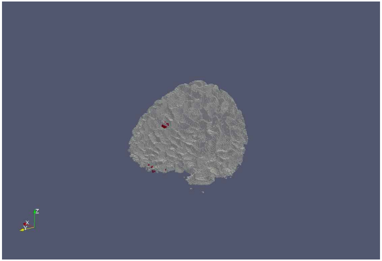

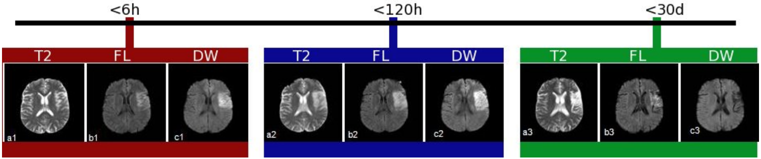

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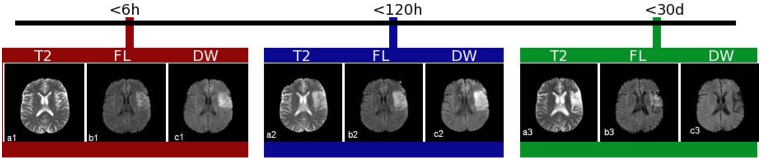

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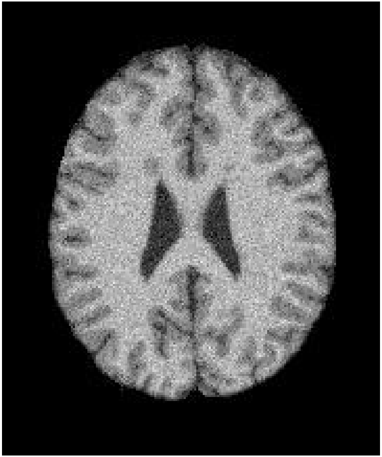

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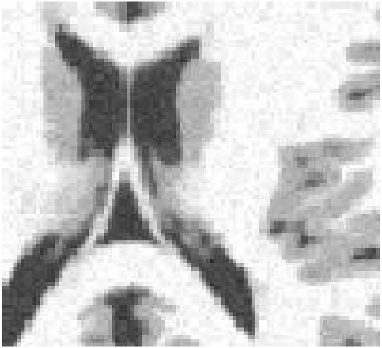

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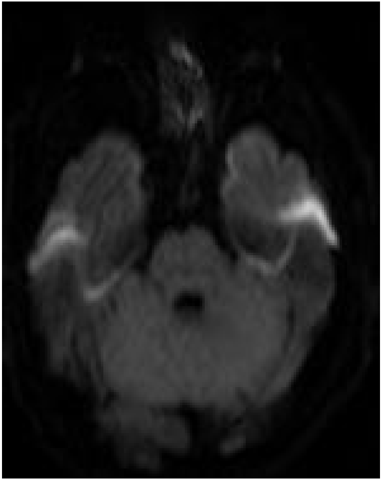

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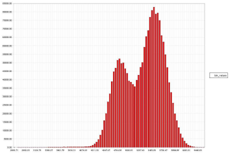
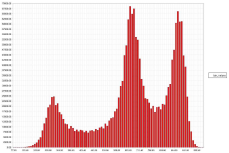


PD



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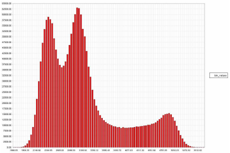

T2

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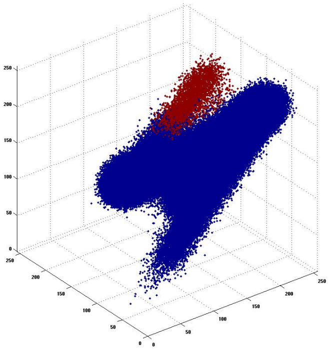

Figure: Feature Space, Lesion in Red

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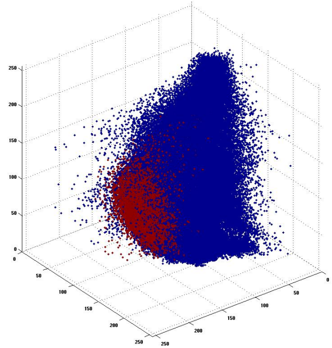

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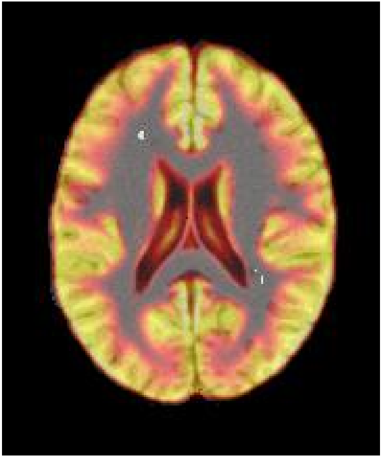

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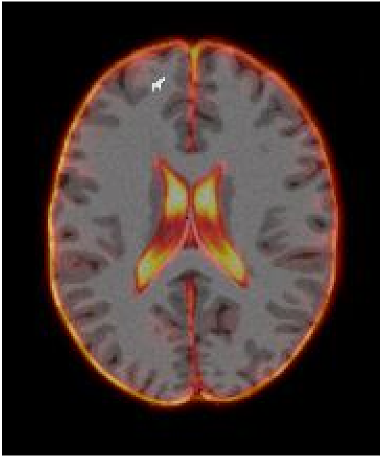

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- E-step solved over restricted class of probability distributions, $\tilde{\mathcal{D}}$, s.t. $q(\mathbf{z}, \omega) = q_Z(\mathbf{z})\, q_W(\omega)$ where $q_Z \in \mathcal{D}_Z$ and $q_W \in \mathcal{D}_W$
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Alternating Maximization

- E-step solved over restricted class of probability distributions, $\tilde{\mathcal{D}}$, s.t. $q(\mathbf{z}, \omega) = q_Z(\mathbf{z})\, q_W(\omega)$ where $q_Z \in \mathcal{D}_Z$ and $q_W \in \mathcal{D}_W$
-

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- Use K-L divergence properties to avoid taking derivatives
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\begin{aligned}
\textbf{E-Z:}\quad q_Z^{(r)} &\propto \exp\left(E_{q_W^{(r-1)}}[\log p(\mathbf{z}|\mathbf{y},\mathbf{W};\psi^{(r)})]\right) &(2)\\
\textbf{E-W:}\quad q_W^{(r)} &\propto \exp\left(E_{q_Z^{(r)}}[\log p(\omega|\mathbf{y},\mathbf{Z};\psi^{(r)})]\right) &(3)\\
\textbf{M:}\quad \psi^{(r+1)} &= \arg\max_{\psi\in\underline{\Psi}} E_{q_Z^{(r)} q_W^{(r)}}[\log p(\mathbf{y},\mathbf{Z},W;\psi)]\,. &(4)
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E-Z

- Note $p(\mathbf{z}|\mathbf{y}, \omega; \psi)$ is Markovian with energy

$$H(\mathbf{z}|\mathbf{y}, \omega; \psi) \propto H_{\mathbf{Z}}(\mathbf{z}; \beta) + \sum_{i \in V} \log g(\mathbf{y}_i|\mathbf{z}_i, \omega_i; \phi), (\text{linear in } \omega) \tag{5}$$

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- $E_{q_W^{(r)}}[W_{im}]$ denoted by $\bar{\omega}_{im}^{(r)}$ becomes

$$\bar{\omega}_{im}^{(r)} = \frac{\alpha_{im} + \frac{1}{2}}{\gamma_{im} + \frac{1}{2} \sum_{k=1}^{K} \delta(y_{im}, \mu_{km}^{(r)}, s_{km}^{(r)})\, q_{Z_i}^{(r)}(k)} \qquad (9)$$

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- ϕ updated using

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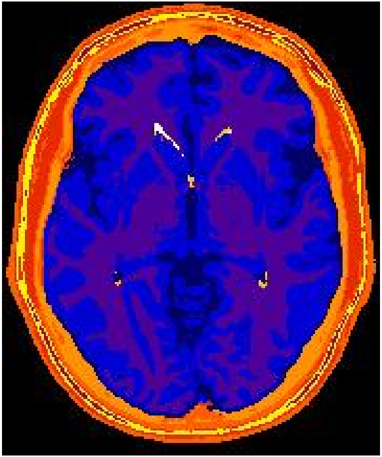

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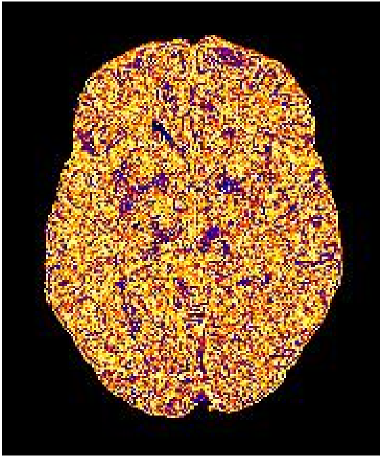

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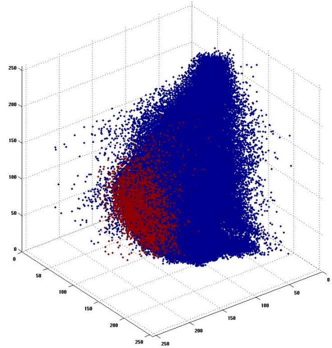

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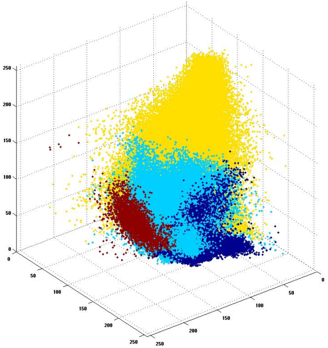


Simulated Data 0% IIH

Method	3%	5%	7%	9%
Mild lesions (0.02% of the voxels)				
AWEM	**68 (+1)**	49 (-21)	**36 (+2)**	**12 (+8)**
[1]	67	**70**	34	0
[3]	56	33	13	4
[2]	52	NA	NA	NA
Moderate lesions (0.18% of the voxels)				
AWEM	**86 (+7)**	80 (-1)	**73 (+14)**	**64 (+27)**
[1]	72	**81**	59	29
[3]	79	69	52	37
[2]	63	NA	NA	NA
Severe lesions (0.52% of the voxels)				
AWEM	**92 (+7)**	86 (-2)	**78 (+6)**	**68 (+27)**
[1]	79	**88**	72	41
[3]	85	72	56	41
[2]	82	NA	NA	NA

Simulated Data 40% IIH

Method	3%	5%	7%	9%
Mild lesions (0.02% of the voxels)				
AWEM	0 (-75)	0 (-65)	0 (-20)	0 (-30)
[1]	**75**	**65**	**20**	**30**
[3]	58	27	13	6
Moderate lesions (0.18% of the voxels)				
AWEM	52 (-24)	51 (-25)	52 (-15)	10 (-38)
[1]	75	**76**	**67**	**48**
[3]	**76**	64	47	31
Severe lesions (0.52% of the voxels)				
AWEM	**87 (+1)**	**84 (+1)**	**77 (+3)**	**66 (+8)**
[1]	75	83	74	58
[3]	86	74	62	45

Real Data (Rennes MS)

	LL	EMS	AWEM
Patient1	0.42	62	**82 (+20)**
Patient2	1.71	54	**56 (+2)**
Patient3	0.29	**47**	45 (-2)
Patient4	1.59	65	**72 (+7)**
Patient5	0.31	**47**	45 (-2)
Average		55 +/-8	60 +/-16

Results

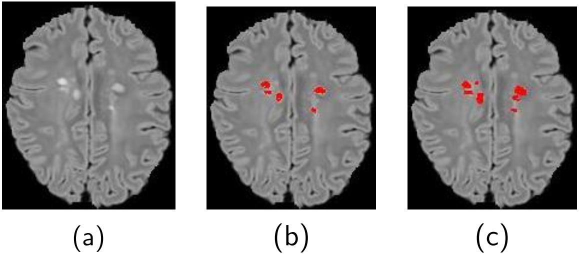
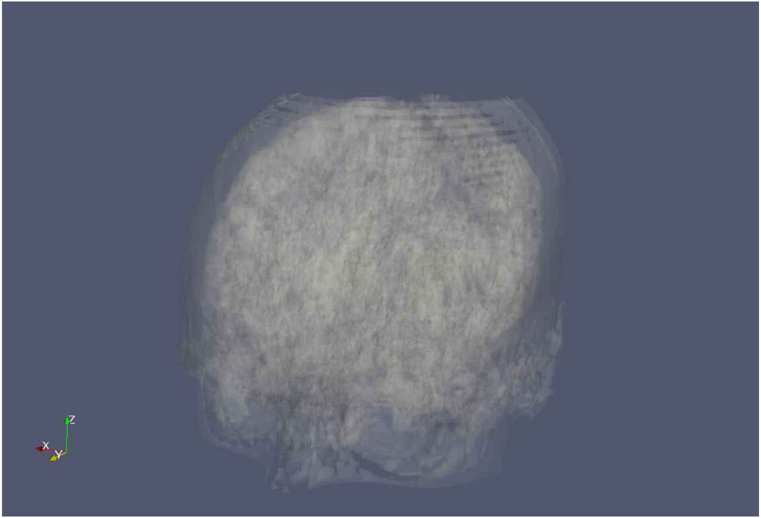


Figure: Real MS data, patient 3. (a): Flair image. (b): identified lesions with our approach (DSC 45%). (c): ground truth .

Results



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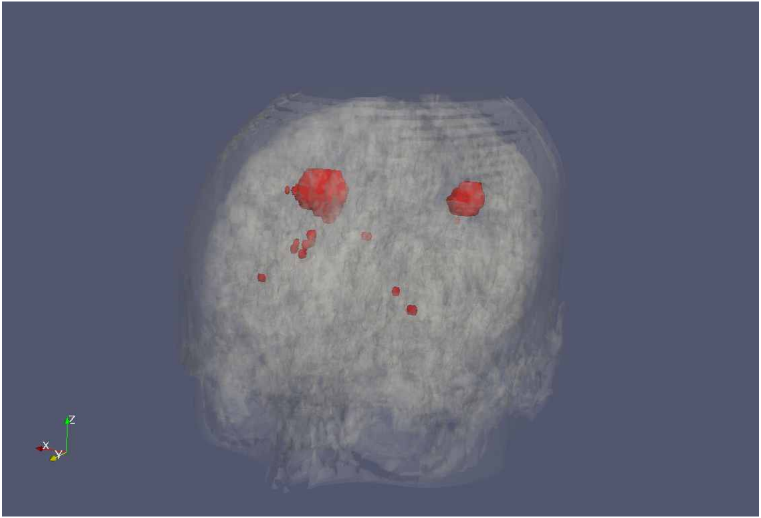

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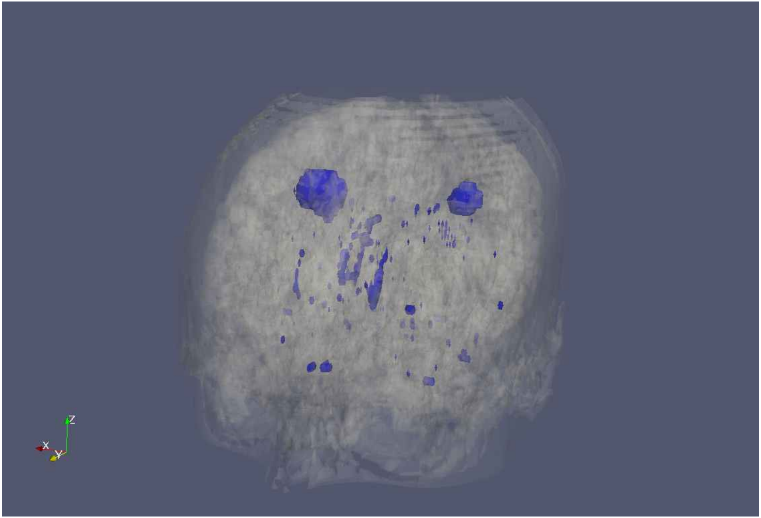

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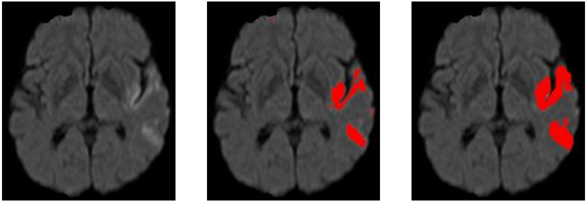

(a) (b) (c)

Figure: Real stroke data. (a): DW image. (b): identified lesions with our approach (DSC 63%). (c): ground truth.

Discussion & Future Work

- Extension to full covariance matrices: temporal multi-sequence data, eg. patient follow-up
- Exploration of Markov Prior
- Other expert weighting schemes, possibly lesion specific
- Extension to handle intensity inhomogeneities
- Sensitivity analysis: initialization, parameter tuning etc. (Darren)
- Evaluation in a semi-supervised context

D. Garcia-Lorenzo, L. Lecoeur, D.L. Arnold, D. L. Collins, and C. Barillot.
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