Contributions to the Chair "Stress Test"

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1 Scientific results

1.1 Publications

Machine learning for extremes

- On the simulation of extreme events with neural networks (with M. Allouche and E. Gobet): `allouche:girard:2026b`

  This article aims at investigating the use of generative methods based on neural networks to simulate extreme events. Although very popular, these methods are mainly invoked in empirical works. Therefore, providing theoretical guidelines for using such models in extreme values context is of primal importance. To this end, we propose an overview of most recent generative methods dedicated to extremes, giving some theoretical and practical tips on their tail behaviour thanks to both extreme-value and copula tools.

- ExceedGAN: simulation above extreme thresholds using Generative Adversarial Networks (with M. Allouche and E. Gobet): `allouche:girard:2026`

  This paper devises a novel neural-inspired approach for simulating multivariate extremes. Specifically, we propose a GAN-based generative model for sampling multivariate data exceeding large thresholds, giving rise to what we refer to as the ExceedGAN algorithm. Our approach is based on approximating marginal log-quantile functions using feedforward neural networks with eLU activation functions specifically introduced for bias correction. An error bound is provided on the margins, assuming a Jth order condition from extreme value theory. The numerical experiments illustrate that ExceedGAN outperforms competitors, both on synthetic and real-world data sets.

- HTGAN: Heavy-Tail GAN for Multivariate Dependent Extremes via Latent-Dimensional Control (with E. Gobet and J. Pachebat): `girard:gobet:2026`

  Models based on transformations of light-tailed noise, such as GANs, fail to capture the tail behaviour of heavy-tailed distributions. In particular, they fail at capturing dependence in extreme regions. We study a modified version of GANs with heavy-tailed input distribution (called HTGAN). Recalling the stable tail dependence function (stdf), a tool from extreme-value theory measuring dependence in extreme regions, we provide a worst-case error bound on the approximation of the stdf of the target with the output of a HTGAN. This bound scales as $N^{-1/(d-1)}$, where N is the dimension of the input noise of the network and d is the dimension of the data. This suggests increasing the dimension of the latent noise to gain precision in the estimation of dependence. We perform experiments, comparing HTGAN with a classical light-tailed GAN (LTGAN) on both synthetic and real datatets exhibiting heavy-tailed characteristics. These experiments confirm our theoretical findings: First, HTGAN is better at reproducing dependence in extremes than LTGAN. Second, the quality of approximation gets better as the dimension of the latent noise increases.

- Learning extreme Expected Shortfall and Conditional Tail Moments with neural networks. Application to cryptocurrency data (with M. Allouche and E. Gobet): `allouche:girard:2025`

  We propose new parameterizations for neural networks to estimate extreme Expected Shortfall, and more generally, extreme conditional tail moments, in heavy-tailed settings as functions of confidence levels. The

proposed neural network estimator is able to extrapolate in the distribution tails thanks to an extension of the usual extreme-value second-order condition to an arbitrary order. The convergence rate of the uniform error between the log-conditional tail moment and its neural network approximation is established. The finite sample performance of the neural network estimator is compared to bias-reduced extreme-value competitors on simulated data. It is shown that our method outperforms them in difficult heavy-tailed situations where other estimators almost all fail. Finally, the neural network estimator is tested on real data to investigate the behavior of cryptocurrency extreme loss returns.

Bayesian methods for extremes

- On the use of a local $\hat{R}$ to improve MCMC convergence diagnostic (with J. Arbel, A. Dutfoy and T. Moins): `moins:arbel:2025`

  Diagnosing convergence of Markov chain Monte Carlo is crucial and remains an essentially unsolved problem. Among the most popular methods, the potential scale reduction factor, commonly named $\hat{R}$, is an indicator that monitors the convergence of output chains to a target distribution, based on a comparison of the between-and within-variances. Several improvements have been suggested since its introduction in the 90s. Here, we aim at better understanding the $\hat{R}$ behaviour by proposing a localized version that focuses on quantiles of the target distribution. This new version relies on key theoretical properties of the associated population value. It naturally leads to proposing a new indicator $\hat{R}^{\infty}$, which is shown to allow both for localizing the Markov chain Monte Carlo convergence in different quantiles of the target distribution, and at the same time for handling some convergence issues not detected by other $\hat{R}$ versions.

Bias reduction for extremes

- A refined extreme quantile estimator for Weibull tail-distributions (with J. El-Methni): `methni:girard:2025`

  We address the estimation of extreme quantiles of Weibull tail-distributions. Since such quantiles are asymptotically larger than the sample maximum, their estimation requires extrapolation methods. In the case of Weibull tail-distributions, classical extreme-value estimators are numerically outperformed by estimators dedicated to this set of light-tailed distributions. The latter estimators of extreme quantiles are based on two key quantities: an order statistic to estimate an intermediate quantile and an estimator of the Weibull tail-coefficient used to extrapolate. The common practice is to select the same intermediate sequence for both estimators. We show how an adapted choice of two different intermediate sequences leads to a reduction of the asymptotic bias associated with the resulting refined estimator. This analysis is supported by an asymptotic normality result associated with the refined estimator. A data-driven method is introduced for the practical selection of the intermediate sequences and our approach is compared to three estimators of extreme quantiles dedicated to Weibull tail-distributions on simulated data. An illustration on a real data set of daily wind measures is also provided.

1.2 Submitted manuscripts

Bayesian methods for extremes

- Approximate Bayesian Computation of reduced-bias extreme risk measures from heavy-tailed distributions (with J. El-Methni): `methni:girard:2025b`

  Most of extrapolation methods dedicated to the estimation of extreme risk measures rely on the approximation of the excesses distribution above a high threshold by a Generalized Pareto Distribution (GPD). We propose an alternative to the GPD, called the Refined Pareto Distribution (RPD), which allows for a second-order approximation of the excesses distribution. The parameters of the RPD are estimated using an Approximate Bayesian Computation (ABC) method, and reduced-bias estimators of extreme risk measures are then derived together with the associated credible intervals. The ABC estimator demonstrates impressive performance over a wide range of heavy-tailed distributions. Its usefulness is also illustrated on two data sets of insurance claims.

Applications of extreme-value theory

- A new family of inequality indices: axioms, inference and tail properties (with J. El-Methni and P. Laveur) `methni:girard:2026`

  Inequality indices provide a quantitative framework for measuring disparity within a distribu- tion, particularly in wealth or income. First, we introduce a unified family of inequality indices that encompasses several classical ones, including Gini, Atkinson, extended Gini, Bonferroni and Mehran indices. Second, it is proved, under appropriate conditions, that indices within this family satisfy six axioms widely accepted in the literature. Third, two general estimators are proposed for this class and their asymptotic normality is established under mild assumptions. Besides, it has been observed that the Gini index is robust to changes in the highest incomes. Leveraging extreme-value theory, we prove a feature shared by the entire family: non-discrimination of tail behaviours in terms of maximum domains of attraction. Notably, this property also holds for several alternatives to the Gini index, including those previously cited. These results are illustrated both on simulated data and on a real income data set.

- Adaptive confidence intervals for extreme quantiles from heavy-tailed distributions (with A. Dutfoy and A. Franchini) `franchini:girard:2026`

  Extreme quantiles are standard risk measures in financial or environmental contexts since they provide information on low-probability but high-impact events. The calculation of accurate confidence intervals on extreme quantiles is then of paramount importance to risk managers. The celebrated Weissman estimator is a simple way to compute such extreme quantiles from heavy-tailed distributions. Asymptotic confidence intervals can also be built based on its asymptotic normality, but they may suffer from poor coverage properties in practice due to oversimplified estimations of the asymptotic bias and variance. We propose several higher order approximations of the Weissman estimator asymptotic distribution together with a data-driven procedure to automatically select the most appropriate one thanks to a supervised classification method. The discrepancy between theoretical and empirical coverages is studied using a new criterion robust with respect to the choice of the number of tail observations involved in the Weissman estimator. The usefulness of the associated adaptive confidence interval is illustrated on an intensive simulation study as well as on two climatic and financial data sets.

Non stationary extremes

- Functional Extreme-PLS (with C. Pakzad) `girard:pakzad:2026`

  We propose an extreme dimension reduction method extending the Extreme-PLS approach to the case where the covariate lies in a possibly infinite-dimensional Hilbert space. The ideas are partly borrowed from both Partial Least-Squares and Sliced Inverse Regression techniques. As such, the method relies on the projection of the covariate onto a subspace and maximizes the covariance between its projection and the response conditionally to an extreme event driven by a random threshold to capture the tail-information. The covariate and the heavy-tailed response are supposed to be linked through a non-linear inverse single-index model and our goal is to infer the index in this regression framework. We propose a new family of estimators and show its asymptotic consistency with convergence rates under the model. Assuming mild conditions on the noise, most of the assumptions are stated in terms of regular variation unlike the standard literature on SIR and single-index regression. Finally, our results are illustrated on a finite-sample study with synthetic functional data as well as on real data from the financial realm, highlighting the effectiveness of the dimension reduction for estimating extreme risk measures.

- Changepoint identification in heavy-tailed distributions (with T. Opitz, A. Usseglio-Carleve and C. Yan) `girard:opitz:2026`

  The problem of detecting the existence of a changepoint in a data sequence and of identifying its position is challenging when the focus is on extreme events and the distribution of data is heavy-tailed. In this setting, we recall several existing techniques, and then we propose a novel robust semi-parametric approach to changepoint identification that does not require the likelihood function. The changepoint is estimated as the position of the maximum of a statistic inspired by classical ANOVA to contrast the

tail behavior of data to the left and right of all changepoint candidates. It is shown that the estimator consistently identifies the true changepoint under mild assumptions when the sample size increases. In numerical experiments, the novel method shows reliable finite-sample behavior for various simulation settings and is very competitive in comparison to the alternative changepoint identification approaches from the literature, especially for small sample sizes. Finally, the utility of the method is highlighted by identifying interpretable changepoints in three real-data applications: very large motor insurance claim amounts for a French administrative region with age as covariate; daily Bitcoin cryptocurrency price data and daily returns of stocks of the Boeing company both with time as covariate.

Proceedings of an Extreme session at "Journées MAS"

- Four contemporary problems in extreme value analysis (with J. El-Methni, J. Legrand, G. Stupfler and A. Usseglio-Carleve) `methni:girard:2026b`

  This article gives a summary of recent results on extreme value analysis that were presented at the Journées MAS 2024 in Poitiers. We first set the general background and motivation for these results, and we then discuss partial solutions to four contemporary problems in extreme value analysis: the construction of bias-reduced estimators of the Expected Shortfall (or Tail-Valueat-Risk) at extreme levels, extremal regression and inference in the presence of dependent data, multivariate inference about extreme quantiles using an analogue of the classical ANOVA method in regression, and improved estimation of tail risk measures using nonparametric resampling of multivariate Generalized Pareto distributions.

1.3 Conferences

- Invited speaker at ICSDS 2025, Seville, Spain `allouche:girard:2025b`
- National conference: Journées de Statistique, Clermont-Ferrand, 2026, `franchini:girard:2026b,guilmaud:methni:2026,laveur:methni:2026`

2 Teaching / training

- Training at BNPP on climate risk quantification methods and tools for finance (3 hours per day during 2 days, september–december, 2025). Focus on measures of extreme risk and dependence. Project coaching on the joint modelling of extreme rainfalls.
- Teaching at Ecole Polytechnique (3rd year) on climate risk: modeling, simulation and estimation (4 hours per day during 3 days, september–december, 2025). Focus on extreme-value statistics, copulas and risk measures.

3 PhD theses supervision

One PhD thesis is half-funded by the Chair:

- Solune Denis (co-advised with Gilles Stupfler, Univ. Angers) "*High-dimensional extreme quantile regression*", Univ. Angers, started on October 1st, 2024.

Two PhD theses with external fundings:

- Antoine Franchini "*Uncertainty quantification associated with extreme quantile estimation*", Univ. Grenoble-Alpes, started on December, 1st, 2024.
- Pearl Laveur (co-advised with Jonathan El-Methni, Univ. Grenoble-Alpes) "*Measures of extreme inequality*", Univ. Grenoble-Alpes, started on October, 1st, 2024.

See Section 5.3 for details on the three associated research projects.

4 Editorial activities

- Associate Editor of *Revstat* since 2019.
- Member of the Advisory Board of *Dependence Modeling* since 2015.

5 Ongoing works & Perspectives 2026–2027

5.1 Teaching / training

- A second training session at BNPP on climate risk quantification methods and tools for finance should take place in 2027.
- The course at Ecole Polytechnique (3rd year) on climate risk: modeling, simulation and estimation should be reconducted in fall 2026.

5.2 PhD thesis supervision

E. Gobet and I are investigating the idea of a new PhD co-supervision on extreme-value statistics and statistical learning. This PhD thesis could be funded by the Chair.

5.3 Research

Machine learning for extremes

- Generative modelling using a space filling curve (with M. Allouche and E. Gobet)

  A space filling curve is a continuous mapping from a multi-dimensional space into a uni-dimensional one. Although it has been discovered more than 100 years ago, it has been used in computer science only recently. We investigate the use of such a curve to build generative models, with a focus on distributions with disjoint supports.

Extreme-value theory

- General asymptotics for extreme value moment-type estimators (with S. Denis, G. Stupfler and A. Usseglio-Carleve)

  We aim at establishing an asymptotic theory for a class of Conditional Tail Moment estimators without restriction on the domain of attraction. The results hinge on a new technical second-order uniform inequality, of independent interest, for powers of a quantile function satisfying a second-order condition.

- Estimation of the tail index for grouped data (with L. Borget, J. El-Methni and P. Laveur)

  In the context of the intership work of L. Borget, we investigate an extension of the Hill estimator of the tail index to the case of grouped data. More generally, this work could be extended to develop inference method for extreme risk measures adapted to grouped data.

Non stationary extremes

- Estimation of extreme quantiles from a mixture of Pareto distributions (with E. Gobet)

  We focus on the situation where the data are drawn from a continuous mixture a Pareto-type distributions with tail-index $\gamma \in [\gamma_1, \gamma_2]$ where $\gamma_1 > 0$. We show that the mixture is still heavy-tailed with tail-index γ_2 and that, in this case, the Hill estimator of the tail-index is strongly biased. A bias correction is introduced thanks to a sharp expansion of the survival distribution of the mixture in the tail. An estimator of extreme quantiles is then derived. Our current work consists in investigating the behavior of this new bias-corrected estimator (i) from the asymptotic point of view, (ii) on simulated data, and (iii) on cryptocurrency data.

- Extreme-PLS for missing data (with C. Pakzad)

  We aim at proposing an extreme dimension reduction method extending the Extreme-PLS approach to the case where some values of the covariate are missing. We also investigate the case of non independent data within the α-mixing framework.

Applications of extremes

- Uncertainty quantification associated with extreme quantile estimation (with A. Franchini)

  The goal of the PhD thesis of Antoine Franchini is to design new confidence intervals on large quantiles using extreme-value theory in the special case of heavy-tailed distributions and to assess the quality of the newly-built intervals.

- Measures of extreme inequality in wealth distribution (with J. El-Methni and P. Laveur)

  The objectives of the PhD thesis of Pearl Laveur are to define and theoretically study a new coefficient for measuring inequalities and dedicated to extreme values. This will enable us to take better account of the behavior of the distribution tail and no longer underestimate inequalities.

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