Contributions to the Chair "Stress Test"

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1 Scientific results

1.1 Publications

Machine learning for extremes

- Estimation of extreme quantiles from heavy-tailed distributions with neural networks (with M. Allouche and E. Gobet): `allouche:girard:2024`

  We propose a new parametrization for one-hidden layer eLU neural networks able to estimate extreme quantiles, starting from heavy-tailed data. We provide an analysis of the uniform error between the extreme log-quantile and its neural network approximation. Numerical experiments are conducted on simulated data to compare the performance of our method with other estimators from the literature. The method is then extended to the estimation of conditional extreme quantiles by combining multi-layers ReLU neural networks with the previous one layer eLU neural network. An illustration is provided on rainfall data in the Cévennes-Vivarais region.

- Learning extreme Expected Shortfall and Conditional Tail Moments with neural networks. Application to cryptocurrency data (with M. Allouche and E. Gobet): `allouche:girard:2025`

  We propose new parameterizations for neural networks to estimate extreme Expected Shortfall, and more generally, extreme conditional tail moments, in heavy-tailed settings as functions of confidence levels. The proposed neural network estimator is able to extrapolate in the distribution tails thanks to an extension of the usual extreme-value second-order condition to an arbitrary order. The convergence rate of the uniform error between the log-conditional tail moment and its neural network approximation is established. The finite sample performance of the neural network estimator is compared to bias-reduced extreme-value competitors on simulated data. It is shown that our method outperforms them in difficult heavy-tailed situations where other estimators almost all fail. Finally, the neural network estimator is tested on real data to investigate the behavior of cryptocurrency extreme loss returns.

Bayesian methods for extremes

- Shrinkage for Extreme Partial Least Squares (with J. Arbel and H. Lorenzo): `arbel:girard:2024`

  This work focuses on dimension-reduction techniques for modelling conditional extreme values. Specifically, we investigate the idea that extreme values of a response variable can be explained by nonlinear functions derived from linear projections of an input random vector. In this context, the estimation of projection directions is examined, as approached by the Extreme Partial Least Squares (EPLS) method–an adaptation of the original Partial Least Squares (PLS) method tailored to the extreme-value framework. Further, a novel interpretation of EPLS directions as maximum likelihood estimators is introduced, utilizing

the von Mises–Fisher distribution applied to hyperballs. The dimension reduction process is enhanced through the Bayesian paradigm, enabling the incorporation of prior information into the projection direction estimation. The maximum a posteriori estimator is derived in two specific cases, elucidating it as a regularization or shrinkage of the EPLS estimator. We also establish its asymptotic behavior as the sample size approaches infinity. A simulation data study is conducted in order to assess the practical utility of our proposed method. This clearly demonstrates its effectiveness even in moderate data problems within high-dimensional settings. Furthermore, we provide an illustrative example of the method's applicability using French farm income data, highlighting its efficacy in real-world scenarios.

- On the use of a local $\hat{R}$ to improve MCMC convergence diagnostic (with J. Arbel, A. Dutfoy and T. Moins): `moins:arbel:2025`

  Diagnosing convergence of Markov chain Monte Carlo is crucial and remains an essentially unsolved problem. Among the most popular methods, the potential scale reduction factor, commonly named $\hat{R}$, is an indicator that monitors the convergence of output chains to a target distribution, based on a comparison of the between-and within-variances. Several improvements have been suggested since its introduction in the 90s. Here, we aim at better understanding the $\hat{R}$ behaviour by proposing a localized version that focuses on quantiles of the target distribution. This new version relies on key theoretical properties of the associated population value. It naturally leads to proposing a new indicator $\hat{R}^{\infty}$, which is shown to allow both for localizing the Markov chain Monte Carlo convergence in different quantiles of the target distribution, and at the same time for handling some convergence issues not detected by other $\hat{R}$ versions.

Bias reduction for extremes

- Reduced-bias estimation of the extreme conditional tail expectation for Box-Cox transforms of heavy-tailed distributions (with M. Allouche and J. El-Methni): `allouche:methni:2024`

  Conditional tail expectation (CTE) is a coherent risk measure defined as the mean of the loss distribution above a high quantile. The existence of the CTE as well as the asymptotic properties of associated estimators however require integrability conditions that may be violated when dealing with heavy-tailed distributions. We introduce Box-Cox transforms of the CTE that have two benefits. First, they alleviate these theoretical issues. Second, they enable to recover a number of risk measures such as conditional tail expectation, expected shortfall, conditional value-at-risk or conditional tail variance. The construction of dedicated estimators is based on the investigation of the asymptotic relationship between Box-Cox transforms of the CTE and quantiles at extreme probability levels, as well as on an extrapolation formula established in the heavy-tailed context. We quantify and estimate the bias induced by the use of these approximations and then introduce reduced-bias estimators whose asymptotic properties are rigorously shown. Their finite-sample properties are assessed on a simulation study and illustrated on real data, highlighting the practical interest of both the bias reduction and the Box-Cox transform.

- A refined extreme quantile estimator for Weibull tail-distributions (with J. El-Methni): `methni:girard:2025`

  We address the estimation of extreme quantiles of Weibull tail-distributions. Since such quantiles are asymptotically larger than the sample maximum, their estimation requires extrapolation methods. In the case of Weibull tail-distributions, classical extreme-value estimators are numerically outperformed by estimators dedicated to this set of light-tailed distributions. The latter estimators of extreme quantiles are based on two key quantities: an order statistic to estimate an intermediate quantile and an estimator of the Weibull tail-coefficient used to extrapolate. The common practice is to select the same intermediate sequence for both

estimators. We show how an adapted choice of two different intermediate sequences leads to a reduction of the asymptotic bias associated with the resulting refined estimator. This analysis is supported by an asymptotic normality result associated with the refined estimator. A data-driven method is introduced for the practical selection of the intermediate sequences and our approach is compared to three estimators of extreme quantiles dedicated to Weibull tail-distributions on simulated data. An illustration on a real data set of daily wind measures is also provided.

Non stationary extremes

- ANOVEX: ANalysis Of Variability for heavy-tailed EXtremes (with T. Opitz and A. Usseglio-Carleve): girard:opitz:2024

  Analysis of variance (ANOVA) is commonly employed to assess differences in the means of independent samples. However, it is unsuitable for evaluating differences in tail behaviour, especially when means do not exist or empirical estimation of moments is inconsistent due to heavy-tailed distributions. Here, we propose an ANOVA-like decomposition to analyse tail variability, allowing for flexible representation of heavy tails through a set of user-defined extreme quantiles, possibly located outside the range of observations. Building on the assumption of regular variation, we introduce a test for significant tail differences among multiple independent samples and derive its asymptotic distribution. We investigate the theoretical behaviour of the test statistics for the case of two samples, each following a Pareto distribution, and explore strategies for setting hyperparameters in the test procedure. To demonstrate the finite-sample performance, we conduct simulations that highlight generally reliable test behaviour for a wide range of situations. The test is applied to identify clusters of financial stock indices with similar extreme log-returns and to detect temporal changes in daily precipitation extremes at rain gauges in Germany.

1.2 Submitted manuscripts

Machine learning for extremes

- On the simulation of extreme events with neural networks (with M. Allouche and E. Gobet): allouche:girard:2024c

  This article aims at investigating the use of generative methods based on neural networks to simulate extreme events. Although very popular, these methods are mainly invoked in empirical works. Therefore, providing theoretical guidelines for using such models in extreme values context is of primal importance. To this end, we propose an overview of most recent generative methods dedicated to extremes, giving some theoretical and practical tips on their tail behaviour thanks to both extreme-value and copula tools.

- Deep generative modeling of multivariate dependent extremes (with E. Gobet and J. Pachebat): girard:gobet:2024

  Dealing with extreme values is a major challenge in probabilistic modeling, important in applications such as economics, engineering and life sciences. Models based on transformations of light-tailed noise, such as GANs, fail to capture the tail behaviour of heavy-tailed distributions. In particular, they fail at capturing dependence in extreme regions. We study a modified version of GANs with heavy-tailed input distribution (called HTGAN). Recalling the stable tail dependence function (stdf), a tool from extreme-value theory measuring dependence in extreme regions, we provide a worst-case error bound on the approximation of the stdf of the target with the output of a HTGAN. This bound scales as $N^{-1/(d-1)}$, where N is the dimension of the input noise of the network and d is the dimension of the data.

This suggests increasing the dimension of the latent noise to gain precision in the estimation of dependence. We perform experiments, comparing HTGAN with a classical light-tailed GAN (LTGAN) on both synthetic and real datatets exhibiting heavy-tailed characteristics. These experiments confirm our theoretical findings: First, HTGAN is better at reproducing dependence in extremes than LTGAN. Second, the quality of approximation gets better as the dimension of the latent noise increases.

Bayesian methods for extremes

- Approximate Bayesian Computation of reduced-bias extreme risk measures from heavy-tailed distributions (with J. El-Methni): `methni:girard:2025b`

  Most of extrapolation methods dedicated to the estimation of extreme risk measures rely on the approximation of the excesses distribution above a high threshold by a Generalized Pareto Distribution (GPD). We propose an alternative to the GPD, called the Refined Pareto Distribution (RPD), which allows for a second-order approximation of the excesses distribution. The parameters of the RPD are estimated using an Approximate Bayesian Computation (ABC) method, and reduced-bias estimators of extreme risk measures are then derived together with the associated credible intervals. The ABC estimator demonstrates impressive performance over a wide range of heavy-tailed distributions. Its usefulness is also illustrated on two data sets of insurance claims.

Proceedings of an Extreme session (see also Paragraph 1.3)

- Four contemporary problems in extreme value analysis (with J. El-Methni, J. Legrand, G. Stupfler and A. Usseglio-Carleve) `methni:girard:2024b`

  This article gives a summary of recent results on extreme value analysis that were presented at the Journées MAS 2024 in Poitiers. We first set the general background and motivation for these results, and we then discuss partial solutions to four contemporary problems in extreme value analysis: the construction of bias-reduced estimators of the Expected Shortfall (or Tail-Valueat-Risk) at extreme levels, extremal regression and inference in the presence of dependent data, multivariate inference about extreme quantiles using an analogue of the classical ANOVA method in regression, and improved estimation of tail risk measures using nonparametric resampling of multivariate Generalized Pareto distributions.

1.3 Conferences

- Organization of a "Computational and statistical methods for extremes in finance" session at the *International Conference on Computational Finance* (ICCF), Amsterdam, april 2024.
- Organization of an "Extreme" session at the "*Journées MAS de la SMAI*", Poitiers, august 2024, with an invitated talk `methni:girard:2024b`.
- Chair of the "Extreme and risk" session at the "*Journées de Statistique de la SFdS*", Bordeaux, may 2024.
- Keynote speaker at the "*Journées de Statistique de la SFdS*", Bordeaux, may 2024.
- International conferences:
  SIAM Conference on Uncertainty Quantification `allouche:girard:2024f,allouche:gobet:2024`,
  ICCF `allouche:girard:2024e,pachebat:girard:2024`, ISNPS `pakzad:girard:2024`,
  EcoSta `usseglio:girard:2024`, EVAN `dkengne:girard:2024`, MathFinance `allouche:girard:2024g`.
- International workshops: `allouche:girard:2024d`, `franchini:dutfoy:2024, laveur:methni:2024`.

2 PhD theses supervision

One PhD thesis is founded by the Chair:

- Jean Pachebat (co-advised with Emmanuel Gobet, Ecole Polytechnique) "*How AI models can deal with extreme values? Application to risk assessment*", Institut Polytechnique de Paris, started on February 1st, 2023. I stopped to co-advise the thesis during summer 2024.

One PhD thesis is half-funded by the Chair:

- Solune Denis (co-advised with Gilles Stupfler, Univ. Angers) "*High-dimensional extreme quantile regression*", Univ. Angers, started on October 1st, 2024.

Two PhD theses with external fundings:

- Antoine Franchini "*Uncertainty quantification associated with extreme quantile estimation*", Univ. Grenoble-Alpes, started on December, 1st, 2024.
- Pearl Laveur (co-advised with Jonathan El Methni, Univ. Grenoble-Alpes) "*Measures of extreme inequality*", Univ. Grenoble-Alpes, started on October, 1st, 2024.

3 Editorial activities

- Associate Editor of *Revstat* since 2019.
- Member of the Advisory Board of *Dependence Modeling* since 2015.

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